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Inverse Domination and Inverse Total Domination in Digraphs

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Abstract
Let D=(V, A) be a digraph. A subset S of V is called a dominating set of D if for every vertex v in V – S, there exists a vertex u in S such that (u, v)∈A. A subset S of V is called a total dominating set of D if S is a dominating set of D and the induced subdigraph ⟨S⟩ has no isolated vertices. In this paper, we introduce the inverse domination parameters corresponding to domination and total domination in digraphs and establish some results on these parameters. Also we introduce the disjoint domination parameters corresponding to domination and total domination in digraphs.

Keywords
Digraph, domination, inverse domination, disjoint domination, total domination, inverse total domination, disjoint total domination.
Mathematics Subject Classification: *05C.*

I. Introduction

In this paper, D=(V, A) is a finite, directed graph with neither loops nor multiple arcs (but pairs of opposite arcs are allowed) and G=(V, E) is a finite, undirected graph with neither loops nor multiple edges. For basic terminology, we refer to Chartrand and Lesniak [2].

A set S of vertices in a graph G=(V, E) is a dominating set if every vertex in V – S is adjacent to some vertex in S. The domination number γ(G) of G is the minimum cardinality of a dominating set of G. Recently many new domination parameters are given in the books by Kulli [8, 9, 10].

The first paper on the inverse domination in graphs was published by Kulli and Sigarkanti [17] and was studied by several graph theorists in [4, 5, 7, 12, 13, 15, 16, 18].

The concept of inverse domination in graphs is as follows:

Let S be a minimum dominating set of G. If V – S contains a dominating set S ' of G, then S ' is called an inverse dominating set with respect to S. The inverse domination number $\surd^{1}$(G) of G is the minimum cardinality of an inverse dominating set of G.

Let G = (V, E) be a graph without isolated vertices. A dominating set S of V is called a total dominating set of G if the induced subgraph ⟨S⟩ has no isolated vertices. The total domination number γ_t(G) of G is the minimum cardinality of a total dominating set of G, [3].

Kulli and Iyer [15] introduced the concept of inverse total domination in graphs as follows:

Let S be a minimum total dominating set of G. If V – S contains a total dominating set S' of G, then S' is called an inverse total dominating set with respect to S. The inverse total domination number of G is the minimum cardinality of an inverse total dominating set of G.

Let D = (V, A) be a digraph. For any vertex u∈V, the sets O(u) = {v/(u, v)∈A} and I(u) = {v/(v, u)∈A} are called the outset and inset of u. The indegree and outdegree of u are defined by id(u)=|I(u)| and od(u)=|O(u)|.

A set S of vertices in a digraph D=(V,A) is a dominating set if for every vertex u in V – S, there exists a vertex v in S such that (v, u) ∈A. The domination number ∈(D) of D is the minimum cardinality of a dominating set of D.

Let D=(V, A) be a digraph in which id(v)+od(v) > 0 for all v ∈ V. A subset S of V is called a total dominating set of D if S is a dominating set of D and the induced subdigraph ⟨S⟩ has no isolated vertices. The total domination number γ_t(D) of D is the minimum cardinality of a total dominating set of D, [1].

In [11], Kulli introduced the maximal domination number and maximal total domination number in digraphs and obtained some results on these parameters.

In this paper, we introduce the analog of inverse domination and inverse total domination in digraphs and also we introduce the analog of disjoint domination and disjoint total domination in digraphs. We obtain several results on these parameters.

II. Results

The concept of inverse domination can be extended to digraphs.

Definition 2.1

Let D = (V, A) be a digraph. Let S be a minimum dominating set in a digraph D. If V – S contains a dominating set S' of D, then S' is called an inverse dominating set with respect to S. The minimum cardinality of an inverse dominating set of a digraph D is called the inverse domination number of D and is denoted by $\surd^{-1}$(D).

Remark 2.2

We note that not all digraphs have inverse dominating sets.

Example 2.3

For the digraph D shown in Figure 1, S = {4, 5} is a minimum dominating set and V – S = {1, 2, 3, 6, 7} is not a dominating set. Thus V – S does not contain a dominating set. Hence the digraph D has no an inverse dominating set.

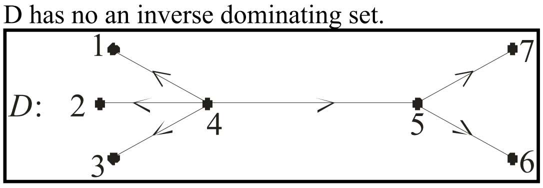

Fig.1:

Definition 2.4

The upper inverse domination number ⌈$^{-1}$(D) of a digraph D is the maximum cardinality of an inverse dominating set of D.

Example 2.5

Let D be the digraph as in Figure 2. The minimum dominating sets of D are {1, 3}, {2, 5} and the corresponding inverse dominating sets are {2, 5}, {1, 3} respectively. Thus γ(D)=2 and γ–1(D) =⌈–1(D)=2. Hence γ(D)=γ–1(D).

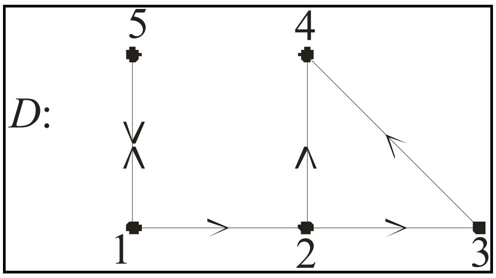

Fig. 2:

Example 2.6

Let D be the digraph as in Figure 3. The only minimum dominating set of D is {1, 3} and the corresponding inverse dominating set is {2, 4, 5}. Therefore γ(D) = 2 and γ^{-1}(D) = Γ^{-1}(D) = 3. Thus γ(D) < γ^{-1}(D).

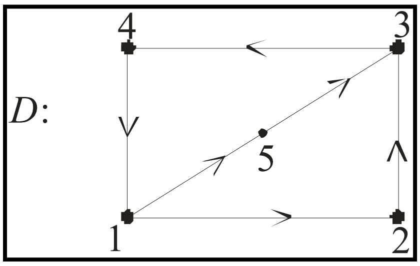

Fig. 3:

Proposition 2.7

For any directed cycle C_{2p}, p≥2,
$\sqrt{-1}$(C2p) = p.

A $\sqrt{}^{-1}$-set is a minimum inverse dominating set of a digraph D.

Proposition 2.8

If a digraph D has a $\sqrt{}^{-1}$-set, then
γ(D) ≤$\sqrt{}^{-1}$(D) (1)
and this bound is sharp.
Proof: Clearly every inverse dominating set of a digraph is a dominating set. Thus (1) holds.
The directed cycles C_{2p}, p ≥ 2 achieve this bound.

Proposition 2.9

If a digraph D has a $\sqrt{}^{-1}$-set, then
γ(D) + $\sqrt{}^{-1}$(D) ≤p (2)
and this bound is sharp.
Proof: (2) follows from the definition of $\sqrt{}^{-1}$(D).
The directed cycles C_{2p}, p≥2 achieve this bound.

II. Disjoint Domination in Digraphs

We introduce the notion of disjoint domination in digraphs.

Definition 3.1

The disjoint domination number γγ(D) of a digraph D is defined as follows: γγ(D) = min {$|S_1| + |S_2|$: S_1 and S_2 are disjoint dominating sets of D}. We say that two disjoint dominating sets, whose union has cardinality γγ(D), is a γγ-pair of D.

Theorem 3.2

For every digraph D with $\sqrt{}^{-1}$-set,
2γ(D) ≤γγ(D) ≤γγ(D) + $\sqrt{}^{-1}$(D) ≤p.

Remark 3.3.

We note that not all digraphs have a disjoint domination number.

Definition 3.4

A digraph D is calledγγ-minimum if γγ(D) ≤2γ□(D). Similarly a digraph D is called γγ- maximum if γγ(D) =p.
For the digraph D of Figure 2, γγ(D)=2γ(D). Thus the digraph D is γγ-minimum.

For the digraph D of Figure 3, γγ(D)=5. Thus the digraph D is γγ- maximum.
For any directed cycle C_{2p}, p≥2, γγ(C_{2p}) = 2γ(C_{2p}) = p. Thus the directed cycles C_{2p}, p ≥2 are γγ-minimum and γγ-maximum.

IV. Inverse Total Domination in Digraphs

The concept of inverse total domination can be extended to digraphs.

A. Definition

Let D = (V, A) be a digraph in which id(v) + od(v) > 0 for all v∈V. Let S be a minimum total dominating set in a digraph D. If V – S contains a total dominating set S' of D, then S' is called an inverse total dominating set with respect to S. The inverse total domination number of D is the minimum cardinality of an inverse total dominating set of D.

B. Definition

The upper inverse total domination number of a digraph D is the maximum cardinality of an inverse total dominating set of D.

C. Example

Let D be a digraph as in Figure 4. The minimum total dominating sets of D are {1, 2, 5} and {3, 4, 6} and the corresponding inverse total dominating sets are {3, 4, 6} and {1, 2, 5} respectively. Therefore γ_t(D) = γ_t^{-1} (G) = Γ_t^{-1} (D) = 3

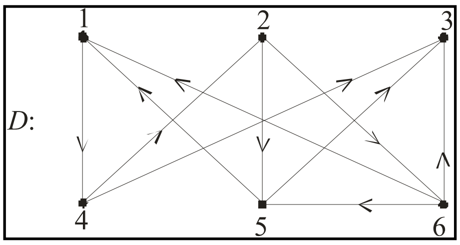

Fig. 4:

A γ_t^{-1} -set is a minimum inverse total dominating set of a digraph D. Not all digraphs without isolated vertices have a total dominating set. We also note that not all digraphs without isolated vertices have an inverse total dominating set. For example, the directed cycle C_4 has a total dominating set, but has no an inverse total dominating set.

D. Proposition

If a digraph D has a -set, then
γ_t(D)γ_t^{-1} (3)
and this bound is sharp.
Proof: Clearly, every inverse total dominating set is a total dominating set. Thus (3) holds.
The digraph D of Figure 4 achieves this bound.

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E. Proposition

If a digraph D has a γ_t^{-1} -set, then

$$\gamma^t(D) \le \gamma_t^{-1}(D) \le p \quad (4)$$

and this bound is sharp.

Proof: (4) follows from the definition of .

The digraph D of Figure 4 achieves this bound.

F. Proposition

Let S be a γ_t-set of a connected digraph D. If a -set exists in D, then D has at least 4 vertices.

Proof: Let S be a γ_t-set of D. Since D has no isolated vertices, $\gamma_t(D) = |S| \ge 2$. If a γ_t^{-1}-set exists, then V – S contains a total dominating set with respect to D. Thus $|V – S| \ge 2$. Thus D has at least 4 vertices.

G. Theorem

If a digraph D has a γ_t-set, then

$2 \le \sqrt{}^{-1t}(D) \le p – 2$.

Proof: By definition, $\gamma_t(D) \ge 2$ and by Proposition 4.4, $\gamma t(D) \le \gamma_t^{-1}$. Thus $2 \le \gamma_t^{-1}(D)$.

By Proposition 4.5,

$$\gamma_t^{-1}(D) \le p – \gamma_t(D)$$
$$\le p – 2, \text{ since } 2\ \gamma_t(D).$$

We establish a Nordhaus-Gaddum type result.

H. Theorem

Let D be a digraph such that both D and have no isolated vertices. Then

$$4 \le g_t^{-1}(D) + g_t^{-1}(\overline{D}) \le 2(p-2),$$

$$4 \le g_t^{-1}(D).g_t^{-1}(\overline{D}) \le (p-2)^2$$

.

Proof: This follows from Theorem 4.7.

V. Disjoint Total Domination in Digraphs

We introduce the notion of disjoint total domination in digraphs.

Definition 5.1. The disjoint total domination number $\gamma_t\gamma_t(D)$ of a digraph D is defined as follows: $\gamma_t\gamma_t(D) = \min \{|S1| + |S2| : S1$ and S2 are disjoint total dominating sets of D$\}$. We say that two disjoint total dominating sets whose union has cardinality $\gamma_t\gamma_t(D)$, is a $\gamma_t\gamma_t$-pair of D.

Theorem 5.2. For every digraph D with g_t^{-1} -set,

$2\ \gamma_t(D) \le \gamma_t\gamma_t(D) \le \gamma_t(D) + \gamma_t^{-1}(D) \le p$.

Remark 5.3. We note that not all digraphs have a disjoint total domination number.

Definition 5.4. A digraph D is called $\gamma_t\gamma_t$-minimum if $\gamma_t\gamma_t(D) \le 2\gamma_t(D)$. Similarly a digraph D is called $\gamma_t\gamma_t$-maximum if $\gamma_t\gamma_t(D)=p$.

For the digraph D of Figure 4, $\gamma_t\gamma_t(D)=6$. Thus the digraph D is $\gamma_t\gamma_t$-maximum and also it is $\gamma_t\gamma_t$-minimum.

VI. Open Problems

The following are some problems for further investigation.

A. Problem

Obtain a necessary and sufficient condition for the existence of an inverse dominating set in a digraph D.

B. Problem

Characterize digraph D for which $\gamma(D) = \sqrt{}^{-1}(D)$.

C. Problem

Characterize digraphs D for which $\gamma(D) + \sqrt{}^{-1}(D) = p$.

D. Problem

Characterize the class of $\gamma\gamma$-minimum digraphs.

E. Problem

Characterize the class of $\gamma\gamma$-maximum digraphs.

F. Problem

Derive a necessary and sufficient condition for the existence of an inverse total dominating set in a digraph D.

G. Problem

Characterize digraphs D for which $\gamma_t(D) = g_t^{-1}(D)$.

H. Problem

Characterize digraphs D for which $\gamma_t(D) + g_t^{-1}(D) = p$.

I. Problem

Characterize the class of $\gamma_t\gamma_t$ -minimum digraphs.

J. Problem

Characterize the class of $\gamma_t\gamma_t$ -maximum digraphs.

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