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Estimating The Optimal Number of Patients of External Clinics, For A Private Hospital, Via Mathematical Programming

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Abstract

There is a great need for administrators of private organizations to be familiar with efficient methods of allocating scarce resources as an organization seeking for profit. In the other side they are seeking for quality which can be measured in terms of several criteria. In the health care service quality output can be measured in terms of, duration of stay, number of procedures, number of admissions or discharges. It is known that high quality is often associated with high payment (cost), although costs must be kept within certain limits. The main objective of the study is to apply the multi objective programming(MOP)with the aid of queuing theory to determine the optimal number of patients to be served via the external clinics of a private hospital so that the decision maker can efficiently , allocate the scarce resources to realize as high level of quality as possible.

Key Words

Goal Programming , conflicting goals, efficient health care facilities.

Introduction

Health care resources are always limited as compared with the demands for efficient services that are frequently needed by the customers (patients). In the opposite side facilities of a hospital, which are unoccupied or unutilized, are a buffer against the risk of not having the needed facilities when the demand for them increases. The cost of that bufferisthe cost of:

1) Construction, set up
2) Maintaining unoccupied facilities.
3) Staff who is partly idle when not needed.

Besides the ideas of cost reduction and profit maximization, important issueshave to be achieved, especiallyfor private organizations. Both objectives may have equivalent importance while conflicting ones. There are additional objectives such as: Improving market share, relationship with public and producing as high quality as possible.

Goal programming (GP) is developed by Charneset al(1961,1977) to provide the DM`s with the opportunity to satisfy or optimize several diversified goalssimultinuaslly.Lee(1973) has applied model for problems in healthcare fieldshowing the flexibility of choosing priorities of the goals as a great advantage since it permits to easily choose between different choices to function the best one.

Piershalla and Wilson(1989) have presentedsome important methods in patients care delivery systems.McQuarrie et al presented a study to find a range for maximal hospital occupancy rates using various queuing systems. Panayiotopoulos et al(1984) studied a hospital emergency department using a general simulation algorithm via queuing systems .Cooper ,J.K et al presented a mathematical method to estimate the optimal number of beds in the cardiac-care services of a hospital.Chalabi,z et al have developed a two stage stochastic mathematical programming formulation to allocate resources between health care programs when there is an exogenous budget and the parameters of the health care models are variable and uncertain. Epstein et al(2008) provided a general mathematical programming model for health care allocation that allows to incorporate important aspects that are not available if the decision making is made using threshold values of incremental cost-effectiveness ratio alone.

Smith and Daniels et all(1988) noted that a healthcare system`s operational activities are comprised of three phases:

- Long-range capacity decisions involving the acquisition of capital intensive resources.
- Medium-range capacity decisions for work force or equipments.
- Scheduling.

Behner,K.G et all have used economic and statistical analysis to formulate and interpret the relationship between service level and nurse stuffing decisions.Sendi, P et al(2003) examined the implications of indivisibility on the mathematical programming results. Stinnett,A, et al(1996) provided a general mathematical programming framework to accommodate information regarding returns to scale indivisibilities in the patient population, program interdependence and ethical constraints.

The Suggested Model:

External clinics represent a very important preface of any healthcare organization. Determining the optimal number of patients helps the DM`s to efficiently allocate the scarce resources such as physicians, nurses and rooms to realize the conflicting objectives of:

- Minimizing allocation costs.
- Maximizing profit.
- Decreasing wailing times for the patients.
- Keeping human and spatial availability.

2) Goal programming model for determining the optimal number of external patients of a private hospital:

Let:

$C_i$be the number of patients who visit clinic i at shift t and day X_{itd}d,i=1,...,5 t=1,…,6 ,d=1,…,6.
cost of serving one patient at clinici.
M: max. number of patients allowed in the system.
Vi: Theprofit of serving one patient at clinici.
V: min.value of profit targeted by the organization.
P_{itd}: available number of physicians of clinic i at shift t and day d i=1,...,5 t=1,…,6 ,d=1,…,6.
n_{td} : available number of nurses at shift t and day d , t=1,…,6 ,d=1,…,6.
r: available number of rooms for external clinics.
w_{itd}: Average time a patient spends at clinic iat shift t and day d (which is a nonlinear function of the capacity at clinic i ,i=1,… ,5.

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W_i: max. time available for all patient at clinic i(usually 2 hours).
λ_{itd}: arrival rate for clinic iat shift t and day d.
μ_{itd}: service rate for clinic i at shift t and day d.
p_i: probability that a patient of type i will be lost(i.e. probability of reneging).
K_i: max. number of patients of type iallowed to leave the system.
notice that i=5 since we assume the following external clinics:

- Internal medicine clinic
- Ear, nose and throat clinic
- Urology clinic
- Bone and joint clinic
- Dermatology clinic

Themodel goals:

$$\min. \sum_{i=1} \sum_{t=1}^{6} \sum_{d=1}^{6} X_{itd} C_i$$

$$\max. \sum_{i=1} \sum_{t=1}^{6} \sum_{d=1}^{6} X_{itd} v_i$$

$$\min. \sum_{i=1} \sum_{t=1}^{6} \sum_{d=1}^{6} X_{itd} w_{itd}$$

Model constraints:

$$\sum_{i=1} \sum \sum X_{itd} v_i \leq V$$

$$\sum_{i=1} X_{itd} \leq P_{itd} \qquad t=1,\ldots,6 \quad d=1,\ldots,6$$

$$\sum_{i=1} X_{itd} \leq n_{td} \qquad t=1,\ldots,6 \quad d=1,\ldots,6$$

$$\sum_{i=1} X_{itd} \leq r$$

$$\sum_{i=1} \sum_{t=1}^{6} \sum_{d=1}^{6} X_{itd} w_{itd} \leq W_i$$

$$P_r\left(\sum_{i=1} X_{itd}\, p_i \leq K_i\right) \geq \alpha_i$$

$$X_{itd} \geq 0$$

Therefore the final model formulated as a goal programming one is as follows:
Lexico.Min. A=$P_1(d^-_1+d^+_2+d^-_3)$, $P_2(d^+_4 + d_5^{+} + d^+_6 + d^+_7)\}$

s.t.

$$\sum_{i=1}\sum_{t=1}^{6}\sum_{d=1}^{6} X_{itd} \; C_i + d_1^- - d_1^+ = C$$

$$\sum_{i=1}\sum_{t=1}^{6}\sum_{d=1}^{6} X_{itd} \; v_i + d_2^- - d_2^+ = V$$

$$\sum_{i=1}\sum_{t=1}^{6}\sum_{d=1}^{6} X_{itd} \; w_{itd} + d_3^- - d_3^+ = Wi$$

$$\sum_{i=1} X_{itd} + d_4^- - d_4^+ = P_{itd}$$

$$\sum_{i=1} X_{itd} + d_5^- - d_5^+ = n_{td}$$

$$\sum_{i=1} X_{itd} + d_6^- - d_6^+ = r$$

$$\sum_{i=1} X_{itd}\, p_i + d_7^- - d_7^+ = \frac{-\ln \alpha_i}{\lambda_{itd}}$$

$$X_{itd} \geq 0$$

$$0 \leq \alpha_i \leq 1$$

Appendix: calculating the average time a patient stays at clinic I,w_{itd}:
In order to calculate the mean time ,w_{itd}, a patient stay at clinic i,we must know the following three information:

- Dist of inter-arrival time
- Dist. of service time.
- Number of physicians available at clinic i at shift t and day d(which is equivalent to c_{itd} in the following calculations).

Assuming the clinic system is M/M/c_{itd} system and that:

$\rho_{itd} = \frac{\lambda_{itd}}{\mu_{itd}}$ the utilization rate at clinic iat shift t and day d

Therefore:

$$E(t_{itd}) = w_{itd} = \frac{c_{itd}\,\mu_{itd}}{(c_{itd}\,\mu_{itd} - \lambda_{itd})^2}\, \rho_{itd}^{\,c_{itd}}$$

$$\rho_{itd} = \frac{c_{itd} - \rho_{itd}}{c_{itd} + \rho_{itd}} \frac{\rho_{itd}^{\,c_{itd}}}{c_{itd}!} \quad .$$

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