

# A Novel Feedback PD Compensator used with a Third-Order Process

Galal A. Hassaan

Emeritus Professor, Dept. of Mechanical Design and Production,  
Faculty of Engineering, Cairo University, Giza, EGYPT

## Abstract

Compensators are used in place of classical PID controllers for possible achievement of better performance. Unsatisfactory dynamics of some industrial processes represent an engineering problem that has to be solved. Highly oscillating processes and very slow processes are examples.

In this paper a novel feedback PD compensator based is proposed and applied to control a third-order process having unsatisfactory dynamic performance. The process has an 85.6 % maximum percentage overshoot and 230 s settling time. The proposed compensator is capable of controlling the steady-state characteristics of the closed-loop control system and its dynamic characteristics. The advantage of the proposed compensator is its simple structure and tuning using an ISE objective function. It was possible with the proposed compensator to satisfy a system performance with relatively low overshoot , low settling time and accepted phase margin. The study shows that a PD feedforward controller is not suitable for such a process.

## Keywords

Third-order processes – compensators – feedback PD compensator – compensator tuning – improving control system performance.

## I. Introduction

Compensators find wide application in both linear and nonlinear dynamic systems. The design of classical compensators such as lag, lead, lag-lead, PID and pre-filter are investigated in automatic control textbooks [1-5].

Kawada and Sogo (2001) proposed a variable gain PD controller design scheme. The controller parameters are tuned corresponding to the jib length, rope length and jib angle [6]. Patel (2002) derived analytical structures of fuzzy PD controllers. He studied the properties of such PD controllers showing the influence of variable cross-point level on controller performance [7]. Gao (2003) used a set of tools to standardize controller tuning. He compared using a PD and LADRC controllers use with a motion control test bed [8]. Wong and Kapila (2004) presented an approach to perform position control of a DC motor experimental setup via the internet. They used a PD controller structure to control the DC motor test bed from a remote web-client PC [9]. De Luca, Siciliano and Zollo (2005) proposed a PD control with online gravity compensation for regulation tasks of robot manipulators with elastic joints [10].

Wang, Tao, Hong and Cho (2006) presented the use of a PD visual controller for microassembly system to acquire better dynamic response. They applied the fuzzy logic to tune the controller which is a model free method [11]. Zhao and others (2007) used a conventional PD controller with 100 % gain and a STR controller to control a low cost linear switched reluctance motor. Both controllers give good dynamic performance and accurate position tracking [12]. Sato and Kameoka (2008) proposed an adaptive control method of a weigh feeder. They used three different controllers: 1DOF PID, 1DOF PD and 2DOF PD controllers. They designed the controllers on the basis of generalized minimum variance control (GMVC) [13]. Aphiratsakun and Parnichkun (2009) studied the design of a fuzzy based gains tuning of a PD controller for joints positions control of the Asian Institute of Technology's leg Exoskeleton-I. They compared the performance with that of the conventional PD controller [14]. Rahimian and Tavazoei (2010) proposed a scheme for computing the stable regions for fractional-order PI and PD controllers. They studied the effectiveness of the proposed method through simulation results

for two example systems [15].

Allen, Neff and Faloutsos (2011) proposed critically damped PD control strategies to precisely obtain target position and velocity constraints for arbitrary initial conditions [16]. Singh and Yadav (2012) presented comparison of the time specification performance between two type of control (LQR and PD) for a double inverted pendulum system. They determined the control strategy for better performance with respect to pendulum angles and cart position. The use of a simple multi PD controller designed by the theory of pole placement [17]. Kadam and Tiwari (2013) presented a simple method for tuning a PD controller for controlling the depth of an autonomous underwater vehicle. They used the gain margin specification to tune the PD controller [18]. Moraes, Castelan and Moren (2013) proposed a full-order dynamic output feedback compensator for time-stamped network control system. They synthesized compensator gains in terms of linear matrix inequalities [19]. Rao, Raghu and Rajasekaran (2013) designed a feedback controller for a DC-DC boost converter to obtain a constant output values of the feedback controller [20]. Liu and Akasaka (2014) addressed the stabilization problem of linear systems subject to input saturation. They revealed that any linear observer can be used to realize the output feedback stabilization [21]. Zhang, Lam and Xia (2014) studied the design and analysis of output feedback delay compensation controller for network control systems. They used an output feedback strategy to generate the control input packet [22].

## II. Analysis

### Process:

The third-order process considered in this analysis has the transfer function,  $G_p(s)$ :

$$G_p(s) = 0.2 / (s^3 + 1.2s^2 + 0.2s + 0.2) \quad (1)$$

The third-order process under consideration has a time response to a unit step input shown in Fig.1.

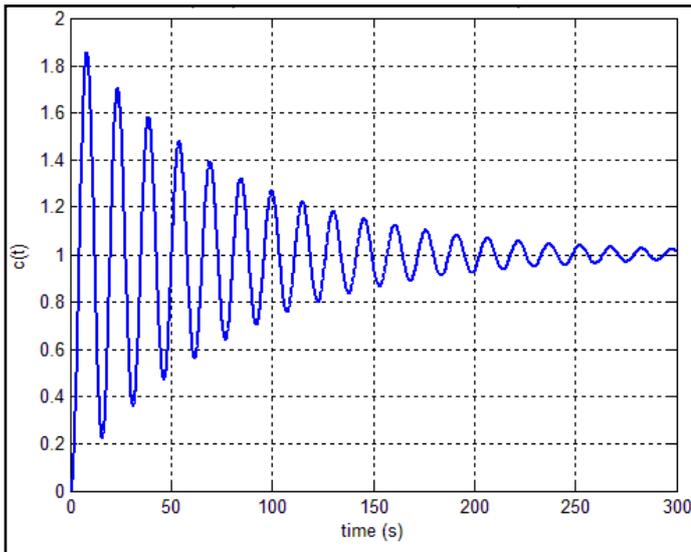


Fig.1: Unit step response of the studied third-order process.

It has the time-based specifications:

- Maximum percentage overshoot: 85.6 %
- Settling time: 230.05 s

**The Proposed Compensator:**

The proposed compensator is a feedback PD compensator. The PD controller is well known as one of the first generation of PID controllers. However, using the PD-structure as a feedback compensator needs investigation which is the purpose of this work. The block diagram of the control system in this case is shown in Fig.2.

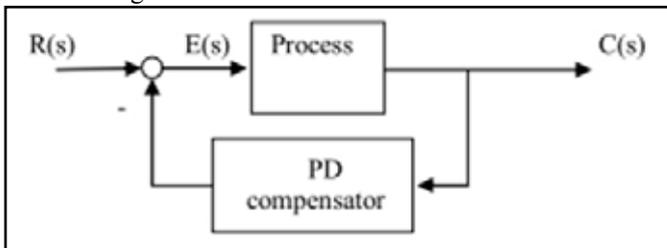


Fig. 2: Block diagram of a feedback compensated control system.

The feedback compensator has a transfer function,  $G_c(s)$  given by:

$$G_c(s) = K_{pc} + K_d s \tag{2}$$

It has the 2 parameters:

- Proportional gain,  $K_{pc}$ .
- Derivative gain,  $K_d$ .

**Control System Transfer Function:**

Using the block diagram of Fig.1, the transfer function of the closed-loop control system is:

$$M(s) = 0.2 / \{s^3 + 1.2s^2 + 0.2(1 + K_d)s + 0.2(1 + K_{pc})\} \tag{3}$$

**System Step Response and Performance:**

A unit step response is generated by MATLAB using the numerator and denominator of Eq. 3 providing the system response  $c(t)$  as function of time for a set of compensator parameters.

The characteristics of the compensated control system quantifying its performance are:

- Steady-state error,  $e_{ss}$ :

Using Eq.3, the system steady-state error for a unit step input is:

$$e_{ss} = K_{pc} / (1 + K_{pc}) \tag{4}$$

- Maximum percentage overshoot,  $OS_{max}$ :

Using the time response of the control system to a unit step input, the maximum percentage overshoot is:

$$OS_{max} = 100 (c_{mas} - c_{ss}) / c_{ss} \tag{5}$$

Where:  $c_{max}$  = maximum time response to a step input.

$c_{ss}$  = steady state response of the control system to the unit step input.

- Settling time,  $T_s$ :

The time response of the system enters a band of  $\pm 5\%$  of the steady-state response and remains inside this band.

**III. Compensator Tuning**

The compensator proposed in this work is tuned using the ISE objective function given by:

$$F = \int (c_{ss} - c)^2 dt \tag{6}$$

Four functional constraints  $c_1, c_2, c_3$  and  $c_4$  are used to control the performance of the control system:

$$c_1 = e_{ss} - e_{ssdes} \tag{7}$$

$$c_2 = OS - OS_{des} \tag{8}$$

$$c_3 = T_s - T_{sdes} \tag{9}$$

$$c_4 = 0.2K_{pc} - 0.24K_d - 0.04 \tag{10}$$

$c_4$  is an stability constraint derived from the Routh-Hurwitz criterion application on the characteristic equation of the closed-loop system.

- Eqs.7-10 are functions of the compensator parameters  $K_{pc}$  and  $K_d$ .

The objective function given by Eq.6 is minimized using the MATLAB optimization toolbox subjected to bounds on the compensator parameters and the functional constraints of Eqs.7 through 10 [23].

**IV. Tuning Results**

MATLAB optimization is used to minimize the objective function (Eq.6) subjected to constraints of the compensator parameters and the functional constraints (Eqs.7-10) with specific desired values for maximum percentage overshoot, settling time and steady-state error.

The tuned compensator parameters are:

$$K_{pc} = 0.022738$$

$$K_d = 2.671408$$

The time response of the compensated control system to a unit step input using the tuned compensator parameters is shown in Fig.3.

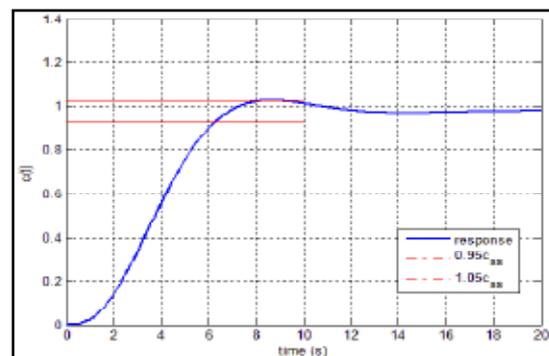


Fig.3 : Unit step response of the feedback PD compensated system.

It has the specifications:

- Maximum percentage overshoot: 4.9983 %
- Settling time: 6.2671 s
- Steady-state error: 0.0222
- Gain margin:  $\infty$  dB
- Phase margin: 63.60 degrees

**V. Why Not A Feedforward PD Controller ?**

Both feedback and feedforward PD compensator and controller have the same PD-structure. So, why not the conventional PD-controller for this process ? If a feedforward PD-controller is used with the third-order process of Eq.1, the closed-loop transfer function becomes:

$$M(s) = (0.2K_d s + 0.2K_{pc}) / \{s^3 + 1.2s^2 + 0.2(1 + K_d) + 0.2(1 + K_{pc})\} \tag{11}$$

Tuning the PD controller using Eq.11 and the same objective function of Eq.6 and functional constraints of Eqs.7-10 yields the following controller parameters:

$$K_{pc} = 59.6316$$

$$K_d = 165.3660$$

The time response of the PD controlled process to a unit step input using the tuned controller parameters is shown in Fig.4.

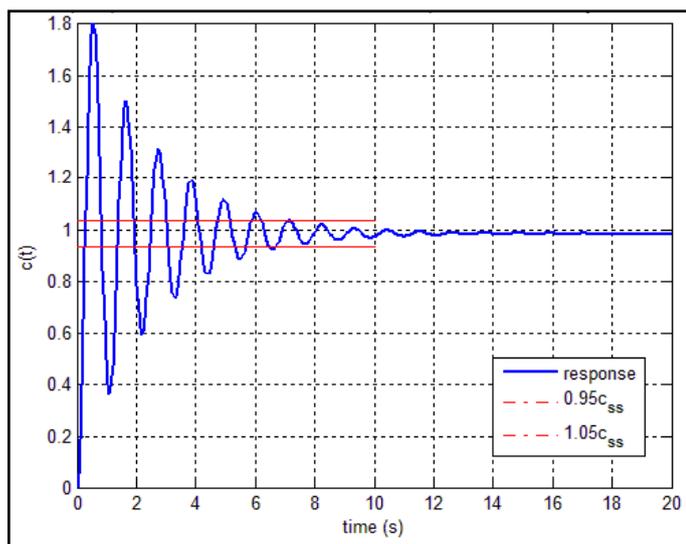


Fig. 4: Unit step response of the feedforward PD controlled system.

It has the specifications:

- Maximum percentage overshoot: 82.84 %
- Settling time: 7.1944 s
- Steady-state error: 0.0164
- Gain margin:  $\infty$  dB
- Phase margin: 8.28 degrees

The performance of the process and the control system using a feedforward controller and a feedback controller is compared in Table 1.

Table 1: Performance comparison

Unit	OS <sub>max</sub> (%)	T <sub>s</sub> (s)	e <sub>ss</sub>	GM (dB)	PM (degrees)
Process	85.600	230.05	-	-	-
Feedback PD compensator	4.998	6.267	0.0222	$\infty$	63.60
Feedforward PD Controller	82.840	7.194	0.0164	$\infty$	8.28

**VI. Conclusions**

- The suggested suggested feedback PD compensator is suitable for controlling a third-order process with difficult dynamic performance.
- Through using the proposed tuning technique for the feedback PD compensator, it was possible to reduce the process maximum percentage overshoot from 85.6 % to only about 5 %.
- Through using the proposed tuning technique for the feedback PD compensator, it was possible to reduce the process settling time from 230 to only about 7.2 s.
- The characteristics of the feedback PD compensated control system were satisfactory since the phase margin of the system was about 64 degrees.
- The study presented in the paper showed that it was not successful to use a feedforward PD controller with the studied thrd-order process. The phase margin was as low as 8.3 degrees indicating the bad performance of the control system.

**References**

[1] J. Dazzo, "Feedback control system analysis and synthesis", McGraw-Hill Education Asia, 1985.

[2] R. Stefani, B. Shahian, C. Savent and G. Hostetter, "Design of feedback control systems", Oxform University Press, 2002.

[3] N. Nise, "Control system engineering", J. Wiley and Sons, 2004.

[4] R. Dorf and R. Bishop, "Modern control systems", Pearson Education Int., 2008.

[5] K. Ogata, "Modern control engineering", Prentice Hall, 2010.

[6] K. Kawada and H. Sogo, "Variable gain PD sway control of a lifted load for a mobile crane", American Control Conference, Proceedings of the 2001 Conference, Vol.2, June 25-27, 2001, pp.953-958.

[7] A. Patel, "Analytical structures and analysis of fuzzy PD controllers with multifuzzy sets having variable cross-point level", Fuzzy Sets & Systems, Vol.129, No.3, August 2002, pp.311-334.

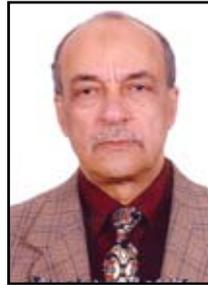
[8] Z. Gao, "Scaling and bandwidth parameterization based controller tuning", Proceedings of Americal Control Conference, 2003, pp.4989-4996.

[9] H. Wong and V. Kapila, "Internet-based remote control of a DC motor using an embedded ethernet microcontroller", Proceedings of the 2004 American Society of Engineering Education Conference and Exposition.

[10] A. De Luka, B. Siciliano and L. Zollo, "PD-control with online gravity compensation for robots with elastic joints:

*Theory and experiments*", *Automatica*, Vol.41, 2005, pp.1809-1819.

- [11] J. Wang, X. Tao, D. Hong and H. Cho, "A fuzzy adaptive PD controller base microassembly system", *Proceedings of SPIE*, Vol.6374, 2006, 63740R, pp.1-10.
- [12] S. Zhao, N. Cheung, W. Gan, J. Yang and J. Pan, "A self tuning regulator for the high precision position control of a linear switched reluctance motor", *IEEE Trans. On Industrial Electronics*, Vol.54, No.5, October 2007, pp.2425-2434.
- [13] T. Sato and K. Kameoka, "Self-tuning type 2 PID control system and its applications", *Proceedings of the 17<sup>th</sup> World Congress, IFAC, Seoul, Korea, 6-11 July, 2008*, pp.11287-11292.
- [14] N. Aphiratsakun and M. Parnichkun, "Fuzzy based gains tuning of PD controller for joint position control of AIT leg exoskeleton-I (ALEX-I)", *International Conference on Robotics and Biomimetics, Bangkok, Thailand, 21-26 February 2009*, pp.859-864.
- [15] M. Rahimian and M. Tavazoei, "Stabilizing fractional-order PI and PD controllers: an integer-order implemented system approach", *Proceedings of the IMechE, Vol.224, Part I: J. of Systems and Control Engineering, 2010*, pp.893-903.
- [16] B. Allen, M. Neff and P. Faloutsos, "Analytic PD control for precise and compliant motion", *Proceedings of the IEEE Int. Conference on Robotics and Automation, May 2011*, pp.6039-6044.
- [17] N. Singh and S. Yadav, "Comparison of LQR and PD controller for stabilizing double inverted pendulum system", *Int. J. of Engineering Research and Development, Vol.1, No.12, July 2012*, pp.69-74.
- [18] S. Kadam and K. Tiwari, "A simplified approach to tune PD controller for the depth control of an autonomous underwater vehicle", *Proceedings of Communication, Computation and Networking Technologies, Vishnupuri Nanded, March 2013, Vol.1*, pp.209-212.
- [19] V. Moraes, E. Castelan and U. Moreno, "Dynamic output feedback compensator for time-stamped networked control systems", *J. Control, Automation and Electrical Systems, Vol.24, No.1-2, April 2013*, pp.22-32.
- [20] G. Rao, S. Raghu and N. Najasekaran, "Design of feedback controller for boost converter using optimization techniques", *International J. Power Electronics and Drive System, Vol.3, No.1, March 2013*, pp.117-128.
- [21] K. Liu and D. Akasaka, "A partial parameterization of nonlinear output feedback controllers for saturated linear systems", *Automatica*, Vol.50, 2014, pp.233-239.
- [22] J. Zhang, J. Lam and Y. Xia, "Output feedback delay compensation control for network control systems with random delays", *Information Sciences, Vol.265, 2014*, pp.156-166.
- [23] P. Vankataraman, "Applied optimization with MATLAB programming", *J. Wiley, 2009*.



### Biography

Prof. Galal Ali Hassaan, Emeritus Professor of System Dynamics and Automatic Control. Now with the Faculty of Engineering, Cairo University, EGYPT. Research on Automatic Control, Mechanical Vibrations and Mechanism Synthesis.