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# Tuning of a First-order Lag-Lead Compensator used with a Simple Pole plus Double Integrator Process

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## Abstract

*Lag-lead compensators are well known in automatic control engineering. They have 3 or 4 parameters to be adjusted (tuned) for proper operation depending on the compensator order. The frequency response of the control system or the root locus plot are traditionally used to tune the compensator in a lengthy procedure.*

*A selected simple pole plus double integrator process is controlled using a first-order lag-lead compensator (through simulation). The lag-lead compensator is tuned by minimizing the sum of square of errors of phase margin plus gain margin of the compensated system using MATLAB. Two functional constrains are used to guarantee the stability of the lag-lead compensated control system. The result was maintaining the gain margin and phase margin of the control system exactly at the desired values. The maximum percentage overshoot and the settling time depends on the desired gain margin. The steady-state error of closed-loop control system using the first-order lag-lead compensator with the studied process is sero. The results are compared with that in a published work.*

## Keywords

*First-order lag-lead compensator - Compensator Tuning - Simple pole plus double integrator process - Control system performance.*

## I. Introduction

Lag-lead compensators can improve the performance of linear control systems through the proper tuning of the compensator parameters. There are two schools in designing lag-lead compensators. One of them uses the frequency response specifications of the compensated control system. The other uses its time response specifications. In the present work we follow the research school using the frequency response specifications. Still the subject is interested to automatic control researchers.

James, Frederick and Taylor (1987) discussed the application of expert system technique to the design of lead-lag compensators for linear SISO systems [1]. Loh, Cai and Tan (2004) studied the auto-tuning of phase lead-lag compensators using the frequency response of the plant using relays with hysteresis [2]. Chang (2004) used phase-lag and phase-lead compensators to control servo control systems [3].

Wang (2006) developed a non-trial-and-error procedure to design lag-lead compensators based on the idea of Yeung-Wang-Chen's graphical-based non-trial-and –error method for 3-parameters lag-lead compensators and Wang's result on the exact and unique solution for single lag and lead compensator design [4]. Zhang, Liu, Dang, Zhang and Ou (2006) used a lag-lead compensator to control asynchronous linear motors for better performance and application to active mass driver control system for vibration control of civil engineering structures [5].

Panda and Padhy (2007) used the genetic algorithm optimization technique to design thyristor controlled series compensator-based controller to enhance the power system stability [6]. Nassirharand (2008) developed an educational software utility for designing linear compensators based on the Youla parameterization technique and an exact model matching criterion [7]. Wang (2009) provided an approach for phase-lead/lag compensators to achieve the desired specifications of gain and phase margins for all-pole stable plant with time-delay [8]. Cao, Watkins and O'Brien (2009) discussed a compensator graphical user interface implemented by MATLAB to enable the user to design a continuous time compensator using the root-locus and Bode plot [9]. Li, Sheng and Chen (2010) derived the impulse response of the distributed order lead-lag compensator using MATLAB and compared with the numerical inverse Laplace Transformation method [10]. Zanasi and Cuoghi (2011) presented three different methods for the synthesis of lead-lag compensator meeting the phase margin and the gain crossover frequency [11]. Nandar (2012) proposed a robot power system stabilizer using genetic algorithm and a first-order lead-lag compensator [12]. Ntogramatzidis, Zanasi and Cuoghi (2012) presented a range of design techniques for the analysis of standard compensators (lead, lag, PID) in terms of the steady-state performance, the stability margins and the crossover frequencies [13].

Nagshi, Rahmani, Vahidi and Hosseinian (2013) introduced a combination of static VAR and a lag-lead controller to enhance the dynamic stability of power systems. They achieved better dynamic performance and reduced steady-state error [14]. Mahmoud (2013) described short steps to design a phase-lead compensator for the mass-spring-damper system to achieve the desired level of its phase margin. He used a first-order phase lead compensator [15]. Hassaan, Al-Gamil and Lashin (2013) used the sum of absolute error criterion to tune a lag-lead compensator used with a first-order process plus an integrator process. Their tuned compensator could reduce the maximum overshoot from 67.3 % to 2.44 % and the settling time from 12 to 0.65 seconds [16]. Bansal and Dewan (2014) investigated the application of PID control a series compensator to stabilize a gimbal system. They were in favor of using a PID controller against using a simple compensator [17]. Sedaghati et. al. (2014) used a 2-stage lesd-lag compensator compared with another types of controllers to provide the damping required to stabilize power systems [18]. Morishita, Suzuki and Iwamoto (2014) described obtaining robustness of a power system stabilizer of a lead-lag compensator using the H∞ control. They optimized the parameters of the compensator using a particle swarm optimization with evolution function considering the closed-loop H∞ norm and a desired response [19].

## II. Analysis

### Process

The process used in the present study is one of the difficult processes consisting of one simple pole and two integrators. This process is unstable either if used in an open-loop fashion, or if used in a unity feedback loop. It has been used by Ogata and it has the transfer function [14]:

$$G_p(s) = 1 / \{s^2(s + 5)\} \qquad (1)$$

The process has the time response to a unit step input shown in Fig.1.

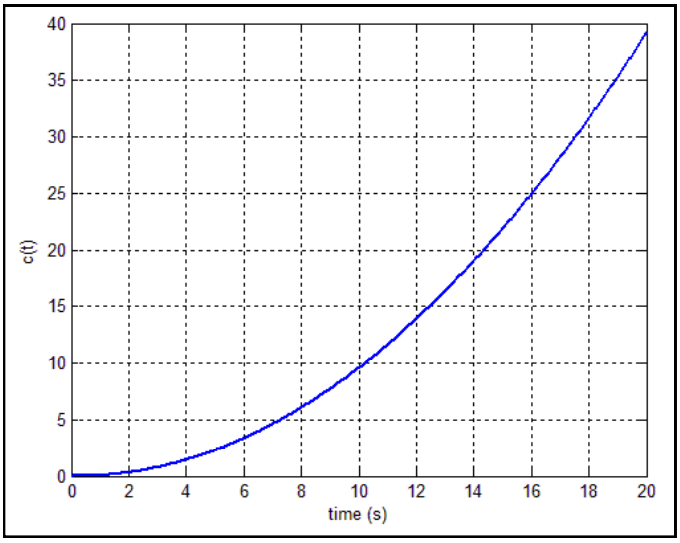

Fig.1: Step response of the process to a unit step input.

The process is completely unstable. Even, if the process is set in a closed-loop unit feedback loop, it will remain unstable.

### Lag-lead First-order Compensator:

The first-order lag-lead compensator has a transfer function $G_c(s)$ given by [14]:

$$G_c(s) = K_c(1 + T_zs) / (1+T_ps) \qquad (2)$$

Where: $T_z$ = the compensator simple zero
$T_p$ = the compensator simple pole
$K_c$ = the compensator gain

The compensator has 3 parameters: $T_z$, $T_p$ and $K_c$ which have to be tuned to satisfy the required specifications of the closed-loop control system defined by:

- The steady-state characteristics.
- The maximum percentage overshoot.
- The maximum percentage undershoot.
- The settling time.

### Control System Transfer Functions:

*Open-loop transfer function:*

The open-loop transfer function is required to assign the relative stability parameters: phase margin and gain margin. It is given from Eqs.1 and 2 for a unit feedback control system incorporating the compensator and process by:

$$G(s)H(s) = K_c(1 + T_zs) / \{s^2(s + 5)(1 + T_ps)\} \qquad (3)$$

### Closed-loop transfer function:

The closed-loop transfer function is required to assign the step response specifications: maximum percentage overshoot, maximum percentage undershoot and settling time. It is given from Eqs.1 and 2 for a unit feedback control system incorporating the compensator and process by:

$$M(s) = (b_0s + b_1) / (a_0s^5 + a_1s^4 + a_2s^3 + a_3s^2 + a_4s + a_5) \qquad (4)$$

where:

$b_0 = K_cT_z$
$b_1 = K_c$
$a_0 = T_zT_p$
$a_1 = T_p + T_z(1 + 5\, T_p)$
$a_2 = 1 + 5T_z + 5T_p$
$a_3 = 5$
$a_4 = K_cT_z$
$a_5 = K_c$

### Frequency-based and Time-based Specifications:

- Eq.3 is used to assign the system gain and phase margins of the control system using the MATLAB command “margin”.
- Eq.4 is used to assign the maximum percentage overshoot and settling time using the MATLAB command “stepinfo”.

## III. Tuning of The First-Order Lag-Lead Compensator

The sum of square of error (ISE) is used as an objective function, F of the optimization process. The error function here is assumed as the sum of the square of difference between the system phase margin and its desired value and difference between the system gain margin and its desired value. That is:

$$F = [PM - PM_{des}]^2 + [GM - GM_{des}]^2 \qquad (5)$$

where PM = system phase margin.
$PM_{des}$ = desired phase margin.
GM = system phase margin.
$GM_{des}$ = desired phase margin.

The compensator parameters have to assigned such that the closed-loop control system is stable. This is achieved through two functional constraints defined from the Routh-Hurwitz criterion of the system. That is:

$$c_1 = a_0a_3 - a_1a_2 \qquad (6)$$

And:

$$c_2 = \alpha_2a_1 - \alpha_1a_3 \qquad (7)$$

$$c_3 = \alpha_1a_5 - \alpha_2\alpha_3 \qquad (8)$$

Where:

$\alpha_1 = (a_1a_2 - a_0a_3) / a_1$
$\alpha_2 = (a_1a_4 - a_0a_5) / a_1$
$\alpha_3 = (\alpha_1a_3 - \alpha_2a_1) / \alpha_1$

### Parameters Limits:

- A lower limit of 0.001 is set for the compensator parameters: $T_z$, $T_p$ and $K_c$.
- An upper limit of 100 is set for the three compensator parameters.

## IV. Tuning Results

The MATLAB command “*fmincon*” is used to minimize the optimization objective function given by Eq.5 subjected to the functional inequality constraints given by Eqs. 6 , 7 and 8 to provide the first-order lag-lead compensator parameters subjected to the limits mentioned in section 6. The results depends on the desired gain margin of the control system. Tables 1 and 2 give the first-order compensator parameters and the control system specifications for a desired phase margin of $50^o$ and a gain margin in the range: 6 – 30 dB.

Table 1: Compensator parameters.

| $GM_{\_desired}$ (dB) | $T_z$ (s) | $T_p$(s) | $K_c$ |
|---|---|---|---|
| 6 | 3.5887 | 0.1159 | 2.6574 |
| 8 | 2.9410 | 0.1417 | 2.2661 |
| 10 | 2.6127 | 0.1284 | 2.1486 |
| 12 | 2.4220 | 0.1112 | 2.1089 |
| 14 | 2.3084 | 0.0937 | 2.1159 |
| 16 | 2.2267 | 0.0788 | 2.1517 |
| 18 | 2.2098 | 0.0731 | 2.0630 |
| 20 | 2.2646 | 0.0722 | 1.8388 |
| 22 | 2.2246 | 0.0638 | 1.8706 |
| 24 | 2.2981 | 0.0634 | 1.6744 |
| 26 | 2.1949 | 0.0535 | 1.8382 |
| 28 | 2.2138 | 0.0506 | 1.7719 |
| 30 | 2.2635 | 0.0497 | 1.6461 |

Table 2: Control system specifications

| $GM_{\_desired}$ (dB) | GM (dB) | PM(deg) | OS (%) | $T_s$ (s) |
|---|---|---|---|---|
| 6 | 6.536 | 49.81 | 24.0534 | 4.5496 |
| 8 | 8 | 50 | 25.6126 | 5.3531 |
| 10 | 10 | 50 | 26.8068 | 5.6850 |
| 12 | 12 | 50 | 27.8083 | 5.8658 |
| 14 | 14 | 50 | 28.2379 | 5.9393 |
| 16 | 16 | 50 | 28.4970 | 5.9400 |
| 18 | 18 | 50 | 28.762 | 6.1198 |
| 20 | 20 | 50 | 29.0504 | 6.5472 |
| 22 | 22 | 50 | 29.0926 | 6.5127 |
| 24 | 24 | 50 | 29.2139 | 6.9351 |
| 26 | 26 | 50 | 29.2856 | 6.6143 |
| 28 | 28 | 50 | 29.304 | 6.7607 |
| 30 | 30 | 50 | 29.383 | 7.0469 |

The time response of the compensated system to a unit step input for a 50° phase margin and a 16 dB gain margin is shown in Fig.2.

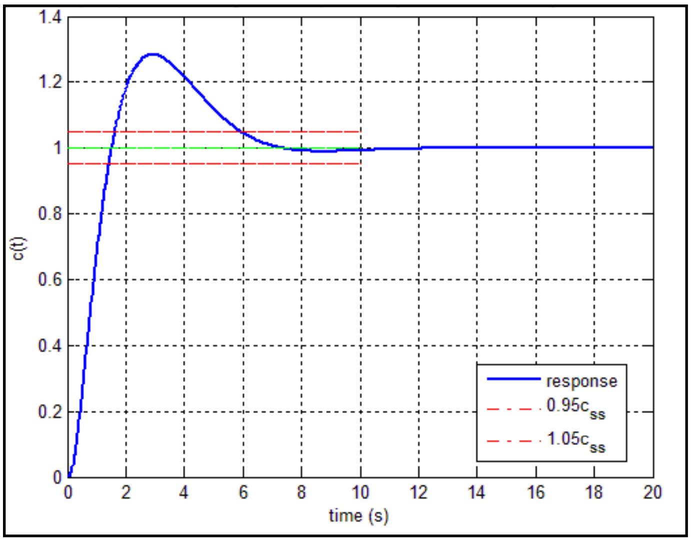

Fig. 2: Step response of the lag-lead compensated system.

## V. Comparison With Ogata Results

Ogata used the frequency response analysis to assign the first-order compensator parameters used to control this unstable process for a 50° desired phase margin and a gain margin > 10 dB [20]. His parameters are:

- $T_z$ = 3.9 s (compared with 2.2267 s in the present study).
- $T_p$ = 0.2557 s (compared with 0.0788 s in the present study).
- $K_c$ = 1.3043 (compared with 2.1517 in the present study).

Those parmameters resulted in a stable control system having the time response to a unit step input shown in Fig.3.

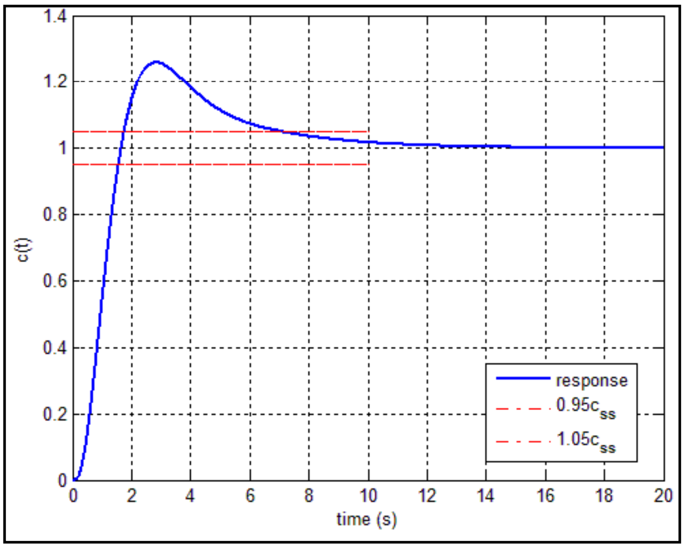

Fig. 3: Step response of the lag-lead compensated system using Ogata parameters.

The control system using Ogata parameters has the performance characteristics:

- Phase margin: 49.9965 degrees
- Gain margin: 16 dB
- Maximum percentage overshoot: 25.82 % (compared with 28.497 % in the present study).
- Settling time: 7.03 s (compared with 5.94 s in the present study).

## VI. Conclusion

- The suggested tuning technique of first-order lag-lead compensators used with an unstable process is simple and efficient.
- Frequency-based control system specifications are used in the tuning technique in conjuction with the MATLAB optimization toolbox.
- Using the proposed tuning technique, it was possible to adjust the settling time to from 4.5 to 7 seconds depending on the desired gain margin.
- The maximum percentage overshoot was in the range 24-29 % depending on the desired gain margin.
- Using the time-based specifications reveals better tuning of the compensator since the time-based specifications will be under control.
- The steady-state error associated with a unit step input was zero.
- The optimization tuning approach used in this work is simple, straight forward, and provides the compensator parameters in a very small time using MATLAB.

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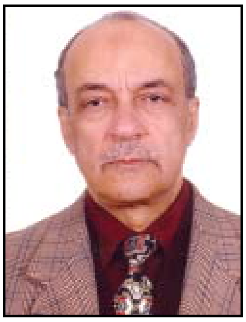

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