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Entire Edge Dominating Transformation Graphs

V. R. Kulli
Dept. of Mathematics, Gulbarga University, Gulbarga, India

Abstract

We introduce four pairs of entire edge dominating transformation graphs G^{xyz} with $x, y, z \in \{+, -\}$. Among these one pair is the entire edge dominating graph G^{+++} introduced by Kulli in [5] and its complement is G^{---}. The remaining pairs with their complements are $\overline{G^{+--}} = G^{--+}$, $\overline{G^{+-+}} = G^{-+-}$, $\overline{G^{++-}} = G^{--+}$. In this paper, we initiate a study of entire edge dominating transformation graphs. Also characterizations are given for graphs for which (i) G^{+++} is complete, (ii) $G^{+++} = E_e d(G)$ and (iii) $G^{+++} = D_e(G)$.

Keywords

Entire Edge Dominating Graph, Semientire Edge Dominating Graph, Transformation. ***Mathematics Subject Classification :*** *05C.*

I. Introduction

By a graph we mean a finite, undirected graph without loops and multiple edges. Any undefined term in this paper may be found in [1]. Unless and otherwise stated, the graphs considered here have $p = |V|$ vertices and $q = |E|$ edges. Let $\overline{G}$ denote the complement of G.

Let $G = (V, E)$ be a graph. A set $F \subseteq E$ is an edge dominating set of G if every edge in $E - F$ is adjacent to some vertex in F. The edge domination number $\gamma'(G)$ of G is the minimum cardinality of an edge dominating set of G. Recently many domination parameters are given in the books by Kulli in [2,3, 4].

An edge dominating set F of G is minimal if every edge $e \in F$, $F - \{e\}$ is not an edge dominating set of G. Let S be the set of all minimal edge dominating sets of G.

The entire edge dominating graph $E_eD(G)$ of a graph $G = (V, E)$ is the graph with the vertex set $E \cup S$ in which two vertices u and v are adjacent if one of the following conditions holds: (i) $u, v \in F$ where F is a minimal edge dominating set in G (ii) $u, v \in S$, $u \cap v \neq \phi$, (iii) $u \in V$, $v \in S$ and $u \in v$. This concept was introduced by Kulli in [5]. Many other graph valued functions in graph theory and domination theory were studied, for example, in [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21].

The edge dominating graph $D_e(G)$ of a graph $G = (V, E)$ is the graph with the vertex set $E \cup S$ in which two vertices u and v are adjacent if $u \in E$ and v is a minimal edge dominating set of G containing u. This concept was introduced by Kulli in [22].

The middle edge dominating graph $M_{ed}(G)$ of a graph $G = (V, E)$ is the graph with the vertex set $E \cup S$ in which two vertices u and v are adjacent if u, v are not disjoint minimal edge dominating sets in G or $u \in E$ and v is a minimal edge dominating set of G containing u. This concept was introduced by Kulli in [23].

The semientire edge dominating graph $E_ed(G)$ of a graph $G = (V, E)$ is the graph with a vertex set $E \cup S$ in which two vertices u and v are adjacent if $u, v \in F$ where F is a minimal edge dominating set in G or u, v are not disjoint minimal edge dominating sets in G or $u \in E$ and v is a minimal edge dominating set in G containing u. This concept was introduced by Kulli in [24].

The common minimal edge dominating graph $CD_e(G)$ of a graph is the graph with vertex set S in which two vertices u and v are adjacent in $CD_e(G)$ if $u, v \in F$ where F is a minimal edge dominating in G. This concept was introduced by Naik in [25].

Recently in [26], Kulli introduced entire total dominating transformation graphs.

II. Results

The definition of the entire edge dominating graph of a graph inspired us to introduce the entire edge dominating transformation graphs in domination theory.

Definition 1. Let $G = (V, E)$ be a graph. Let S be the set of all minimal edge dominating sets in G. Let x, y, z be three variables each taking value + or –. The entire edge dominating transformation graph G^{xyz} is the graph with vertex set $V \cup S$ and for any two vertices u and v in $V \cup S$, u and v are adjacent in G^{xyz} if one of the following conditions holds: (i) $u, v \in E$. $x = +$ if $u, v \in F$ where F is a minimal edge dominating set in G. $x = -$ if $u, v \notin F$, where F is a minimal edge dominating set in G. (ii) $u, v \in S$. $y = +$ if $u \cap v \neq \phi$. $y = -$ if $u \cap v = \phi$. (iii) $u \in E$ and $v \in S$. $z = +$ if $u \in v$. $z = -$ if $u \notin v$.

Using the above entire edge transformation, we get eight distinct entire edge dominating transformation graphs: G^{+++}, G^{+-+}, G^{-++}, G^{+--}, G^{-+-}, G^{--+}, G^{---}, G^{++-}

Example 2: In Figure 1, a graph G and its entire edge dominating transformation graph G^{+++} are shown.

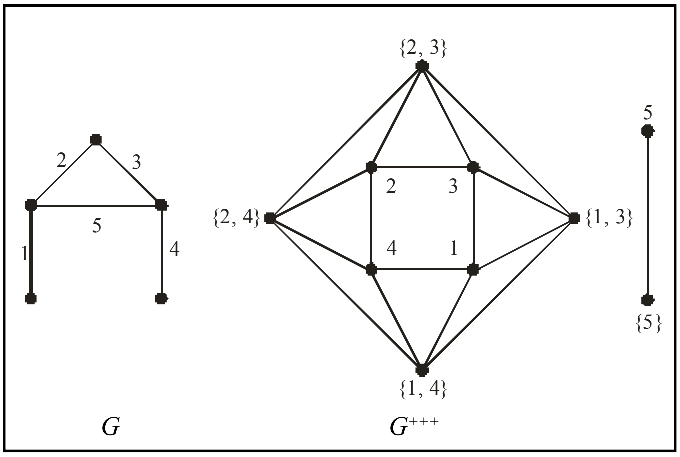

Fig. 1:

Among entire edge transformation graphs one is the entire edge dominating graph G^{+++}. Therefore we have

Proposition 3. For any graph G without isolated vertices, $G^{+++} = E_eD(G)$.

Proposition 4. For any graph G without isolated vertices,

1) $\overline{G^{+++}} = G^{---}$ 2) $\overline{G^{++-}} = G^{--+}$

3) $\overline{G^{+-+}} = G^{-+-}$ 4) $\overline{G^{+--}} = G^{-++}$.

Proof: Each follows from the definition of G^{xyz} and $\overline{G}$.

Remark 5. For any graph G without isolated vertices, $D_e(G)$ is a subgraph G^{+++}.

Remark 6. For any graph G without isolated vertices, $M_ed(G)$ is a subgraph of G^{+++}.

Remark 7. For any graph G without isolated vertices, $E_ed(G)$ is a subgraph of G^{+++}.

Remark 8. For any graph G without isolated vertices, $CD_e(G)$ is an induced subgraph of G^{+++}.

Theorem 9. $G^{+++} = K_{p+1}$ if and only if $G = pK_2$, $p\geq1$.

Proof: Suppose $G = pK_2$, $p\geq1$. Then there exists exactly one minimal edge dominating set F containing all edges of G. If u is the corresponding vertex of F in G^{+++}, then the vertex set of G^{+++} is $E\cup\{u\}$. Clearly G^{+++} has $p+1$ vertices. Since F contains all p edges of G, the corresponding vertices are adjacent in G^{+++}. Thus p vertices together with the vertex corresponding to F form K_{p+1}. Thus $G^{+++} = K_{p+1}$.

Conversely suppose $G^{+++} = K_{p+1}$. We now prove that $G = pK_2$, $p\geq1$. On the contrary, assume $G \neq pK_2$. Then there exist at least two minimal edge dominating sets F_1 and F_2 in G. We consider the following two cases.

Case 1. Suppose $F_1 \cap F_2 = \phi$. Then the corresponding vertices of F_1 and F_2 are not adjacent in G^{+++}, a contradiction.

Case 2. Suppose $F_1\cap F_2 \neq \phi$. Then we have $u \in F_1$ and $u \notin F_2$. Therefore u and F_2 are not adjacent vertices in G^{+++}, which is a contradiction.

From the above two cases, we conclude that G has exactly one minimal edge dominating set.

Hence $G = pK_2$, $p\geq1$.

We present a characterization of graphs whose entire edge dominating graphs are complete.

Theorem 10. For any graph G without isolated vertices, G^{+++} is complete if and only if every component of G is K_2.

Proof: This follows from Theorem 9.

We now characterize graphs G for which $G^{+++} = E_ed(G)$.

Theorem 11. For any graph G without isolated vertices, $E_ed(G) \subseteq G^{+++}$.

Furthermore, equality holds if and only if one of the following conditions holds:

G has exactly one minimal edge dominating set containing all edges.

Every pair of minimal edge dominating sets of G are disjoint.

Proof: By Remark 7, $E_ed(G) \subseteq G^{+++}$.

Suppose $E_ed(G) = G^{+++}$. We prove that G satisfies (i) or (ii). Assume G has at least two minimal edge dominating sets say $D_1, D_2, \ldots, D_n$; $n\geq2$. Suppose $D_1\cap D_2 \neq \phi$. Then the corresponding vertices of D_1 and D_2 are adjacent in G^{+++} and are not adjacent in $E_ed(G)$. Hence $E_ed(G) \neq G^{+++}$, a contradiction. Thus G satisfies (i). Suppose $D_1 \cap D_2 = \phi$. Then the corresponding vertices of D_1 and D_2 are not adjacent in G^{+++}, it implies that every pair of minimal edge dominating sets of G are disjoint. Thus G satisfies (ii).

Conversely suppose G satisfies (i). Then clearly $E_ed(D) = G^{+++}$. Suppose G satisfies (ii). Then two vertices corresponding to minimal edge dominating sets cannot be adjacent in G^{+++}. Thus $G^{+++} \subseteq E_ed(G)$, and since $E_ed(G) \subseteq G^{+++}$, we have $E_ed(G) = G^{+++}$.

Theorem 12. $E_eD(G) = pK_2$ if and only if $G = K_{1,p}$, $p\geq1$ or K_3.

Proof: Suppose $E_eD(G) = pK_2$. We now prove that G is either $K_{1,p}$, $p\geq1$ or K_3. On the contrary, assume $G \neq K_{1,p}$ and $G\neq K_3$. Then there exists at least one minimal edge dominating set F containing at least two edges e_1 and e_2 of G. By definition, the corresponding vertices of F, e_1, e_2, form a subgraph K_3 in G^{+++}, which is a contradiction. Thus $G = K_{1,p}$, $p\geq1$ or K_3.

Conversely suppose $G = K_{1,p}$, $p\geq1$. Then each minimal edge dominating set contains exactly one edge e_i, $1\leq i \leq p$. By definition, it follows that $G^{+++}=pK_2$, $p\geq1$. Now suppose $G = K_3$. Clearly each minimal edge dominating set contains exactly one edge. Hence $G^{+++} = 3K_2$.

We now characterize graphs G for which $G^{+++} = D_e(G)$.

Theorem 13. For any graph G without isolated vertices,

$$D_e(G) \subseteq G^{+++}.$$

Furthermore, $G^{+++} = D_e(G)$ if and only if every minimal edge dominating set contains exactly one edge.

Proof: By Remark 5, $D_e(G) \subseteq G^{+++}$.

We now prove the second part.

Suppose $G^{+++} = D_e(G)$. Let e_1 and e_2 be any two edges of G. Then the corresponding vertices of e_1 and e_2 in $D_e(G)$ are not adjacent. Hence two edges of G are not in the same minimal edge dominating set. Thus every minimal edge dominating set of G contains exactly one edge.

Conversely suppose every minimal edge dominating set of G contains exactly one edge. Then any two edges of G in G^{+++} are not adjacent. Hence $G^{+++} \subseteq D_e(G)$ and since $D_e(G) \subseteq G^{+++}$, it implies that G^{+++} and $D_e(G)$ are isomorphic.

III. Conclusion

In 1932, Whitney introduced a graph valued function line graph. Since then, there are many graph valued functions in graph theory and also in domination theory. In this paper, we introduced the generalization of entire edge dominating graph of a graph and obtained four distinct pairs of entire edge transformation graphs and obtained some properties of the entire edge transformation graph G^{+++}. In graph theory, eulerian graphs and hamiltonian graphs play very important role in several applications, namely, in electric networks, in design of computer systems. So that the existence for an eulerian cycle and a hamiltonian cycle in entire edge dominating transformation graphs is of special interest.

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