

A Design of a Pre-processed Single Tree Search Algorithm in the MIMO Multi Antenna System

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Abstract

In the MIMO multi antenna system, in order to meet the requirements of signal detection algorithm computational complexity and performance, a single tree search soft sphere decoding detection algorithm has been proposed, but this algorithm does not guarantee that its solution sequence is the most efficient solution sequence in the solving process. In view of this, through the study of single tree search soft sphere decoding algorithm, a low complexity pre-processed single tree search algorithm is proposed. The results of the simulation demonstrate that the performance of the pre-processed single search tree algorithm is better than that of the single search tree algorithm, and the complexity of the algorithm is lower than that of the single search algorithm under the case of low SNR.

Key words

MIMO; pre-processed; STS; soft sphere decoding; Signal detection algorithm

I. Introduction

With the continuous development of mobile communication technology, LTE-A has become more mature, many businesses put more focus on the related technology research and development of the next generation of mobile communications, that is, 5G. 5G will use six key technologies: D2D technology, Dense Network, Co-frequency Co-time Full Duplex, High Frequency Transmission, New Network Architecture and a new multi antenna MIMO Technology. The multiple input multiple output (MIMO) technology can greatly improve data transmission rate and transmission quality of the signal without increasing the system bandwidth and transmitting power, and can improve the system capacity and the data transmission reliability. Therefore, the algorithm of the receiver is particularly important.

With the extensive and deep research on the MIMO technology, more and more signal detection algorithms are proposed. There are the zero forcing (ZF) algorithm, the minimum mean square error (MMSE) algorithm, the maximum likelihood (ML) algorithm and the sphere decoding algorithm, which are commonly used. Among them, the ZF or the MMSE algorithm is relatively simple, but the performance of them are not very good; the usage of the ML algorithm in the receiving end can achieve the best performance, but it conducts the blind search instead of using any search strategy, so its computational complexity will increase exponentially with the increase of the number of antennas and the modulation order, therefore, it cannot be applied to the real-time systems; meanwhile, the main problem of the ML decoding algorithm is computational complexity, and the most fundamental method to solve this problem is to reduce the number of the search points. And the sphere decoding algorithm is based on the idea, which uses low computational complexity to obtain the approximate decoding performance of the maximum likelihood algorithm. But the data obtained by the spherical decoding is hard information after demodulation, which is not conducive to the decoding of the Turbo decoding module. However, the soft sphere decoding algorithm, which is different from the spherical decoding algorithm, can solve this problem. It skips the demodulation module and directly enters the Turbo decoding stage. Moreover, it provides the soft information that is more suitable for the handling of Turbo decoding. Therefore, it is necessary to study the soft sphere decoding. However, the traditional soft sphere decoding detection algorithm has very high complexity in realization.

The reference[5] introduces a search method of the soft detection spherical decoding that reduces the leaf nodes, to greatly reduce the complexity of the algorithm under the premise of ensuring the performance. However,

the method cannot guarantee that its solving order is the most efficient solving order in the solution process. Therefore, a preprocessing STS algorithm based on the STS algorithm is proposed in the article, which can guarantee the decoding sequence according to the optimal sequence. In the case of low signal to noise ratio, the algorithm not only improves the decoding performance, but also reduces complexity of the soft sphere decoding algorithm.

II. System model

A wireless MIMO transmission system is designed, with N_T the transmitter has transmitting antennas in the transmitter, N_R receiving antennas ($N_R > N_T$) in the receiving end. Its simulation module is shown in the Figure 1:

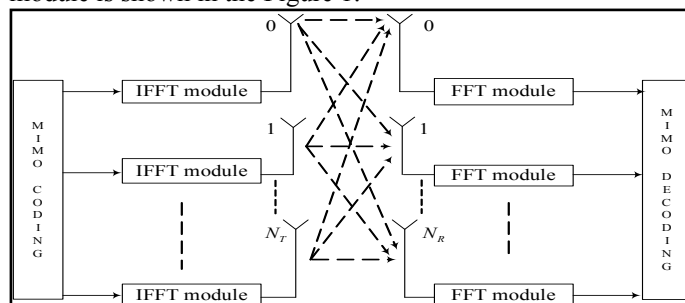


Fig. 1: the MIMO system diagram

The signal received by the receiver can be presented the y , through the Gauss white noise channel with flat Rayleigh fading, the received signal y can be expressed as:

$$y = Hx + n \quad (1)$$

$$y = [y_1, y_2, \dots, y_{N_R}]^T$$

In the formula (1),

$$x = [x_1, x_2, \dots, x_{N_T}]^T$$

is the transmitted signal vector;

$y = [y_1, y_2, \dots, y_{N_R}]^T$ is the received signal vector; $H \in C^{N_R \times N_T}$ is the channel transmission matrix with N_R rows and N_T columns, and satisfies Gauss distribution; $n \in C^{N_R \times 1}$ is the Noise vector and its various components are independent and identically distributed with zero mean and variance of σ^2 Gaussian white noise. Under the premise of the Rayleigh channel and the independent identical

distribution, each element h of \mathbf{H} satisfies the complex Gauss distribution. Assuming that the total power of the transmitter is 1, the average signal to noise ratio(SNR) of each receiving antenna is $1 / \sigma^2$.

This article is for soft spherical decoding detection algorithm in the real number system, so a complex linear system is turned into an equivalent real linear system through real value decomposition. The equation (1) plural form of real value decomposition is shown as the following form:

$$y = \begin{bmatrix} \text{Re}(y) \\ \text{Im}(y) \end{bmatrix} = \begin{bmatrix} \text{Re}(H) & -\text{Im}(H) \\ \text{Im}(H) & \text{Re}(H) \end{bmatrix} \begin{bmatrix} \text{Re}(s) \\ \text{Im}(s) \end{bmatrix} + \begin{bmatrix} \text{Re}(n) \\ \text{Im}(n) \end{bmatrix} \quad (2)$$

$\text{Re}(\bullet)$ and $\text{Im}(\bullet)$ represent the real and imaginary parts of the complex symbols respectively in the equation (2).

III. Soft sphere decoding algorithm

A. Soft information output

Bits stream modulation is mapped as the transmission signal $\mathbf{x} \in \mathcal{O}^{N_T}$ after channel coding in a MIMO system of $N_T \times N_R$, and \mathcal{O}^{N_T} represents N_T dimensional signal vector constituted of Constellation points. The method that receiver calculates the log-likelihood ratio of each encoded bits per symbol is the Max-Log Algorithm. $x_{j,i}$ represents the i -th bits of the j -th symbol in \mathbf{x} , we assume the soft information can be judged by the i -th bit of the j -th symbol as $L(x_{j,i})$, which is defined by a logarithmic likelihood ratio form of A Posterior Probability:

$$LLR(x_{j,i}) = \log \frac{P(x_{j,i} = 0 / y, \mathbf{H})}{P(x_{j,i} = 1 / y, \mathbf{H})} \quad (3)$$

$$= \log \frac{\sum_{x \in x_{j,i}^{(0)}} P(\hat{x}_j = x / y, \mathbf{H})}{\sum_{x \in x_{j,i}^{(1)}} P(\hat{x}_j = x / y, \mathbf{H})}$$

To reduce computational complexity, the use of Max-Log approximation is expressed as:

$$LLR(x_{j,i}) = \max_{x \in x_{j,i}^{(1)}} (-|y - \mathbf{H}x|^2) - \max_{x \in x_{j,i}^{(0)}} (-|y - \mathbf{H}x|^2) \quad (4)$$

$$= \min_{x \in x_{j,i}^{(0)}} (|y - \mathbf{H}x|^2) - \min_{x \in x_{j,i}^{(1)}} (|y - \mathbf{H}x|^2)$$

Where \mathbf{x} is the constellation point of the corresponding modulation form, and represents the set of all symbols containing bits 1, denotes the set of all symbols containing bits 0, and the corresponding symbols of \mathbf{x} and \mathbf{y} are mutually disjointing. One of the two minimum in the formula (4) can be calculated through the formula:

$$\lambda^{ML} = \|y - \mathbf{H}x^{ML}\|^2 \quad (5)$$

Then the other minimum of the formula (4) is defined as:

$$\lambda^{\overline{ML}} = \min_{x \in x_{j,i}^{(x_{j,i}^{\overline{ML}})}} \|y - \mathbf{H}x^{ML}\|^2 \quad (6)$$

The equation (5) and the equation (6) are written as the Piecewise Function form:

$$L(x_{j,i}) = \begin{cases} \lambda^{ML} - \lambda^{\overline{ML}} & x_{j,i}^{ML} = 0 \\ \lambda^{\overline{ML}} - \lambda^{ML} & x_{j,i}^{ML} = 1 \end{cases} \quad (7)$$

From the equation(7), it can be known that using Max-Log maximum to solve the maximum posteriori probability is equivalent to solve the value of $x^{ML}, \lambda^{ML}, \lambda^{\overline{ML}}$.

B. STS algorithm

The process of solving the values of $x^{ML}, \lambda^{ML}, \lambda^{\overline{ML}}$ can be converted to a single search procedure. Firstly, the QR decomposition of the channel matrix $\hat{\mathbf{Y}}$ is conducted, where \mathbf{Q} is a unitary matrix and \mathbf{R} is an upper triangular matrix. Then the can be obtained through using that \mathbf{Q}^H left multiplies the received vector \mathbf{Y} :

$$\hat{\mathbf{Y}} = \mathbf{Q}^H \mathbf{Y} = \mathbf{Q}^H (\mathbf{Q} \mathbf{R} \mathbf{X} + \mathbf{N}) = \mathbf{R} \mathbf{X} + \mathbf{Q}^H \mathbf{N} \quad (8)$$

To gain a more intuitive understanding of the process of QR decomposition algorithm, equation (8) is represented by matrix multiplication form:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n_R} \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1n_T} \\ 0 & R_{22} & \cdots & R_{2n_T} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_{n_R n_T} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n_R} \end{bmatrix} + \mathbf{W}$$

(1.9)

among them, $\mathbf{W} = \mathbf{Q}^H \mathbf{N}$.

From the formula (8), $\mathbf{Q}^H \mathbf{N}$ has the same statistical distribution as the noise, then calculating $\lambda^{ML}, \lambda^{\overline{ML}}$ can be equivalent to:

$$\lambda^{ML} = \min_{x \in \mathcal{O}^{N_T}} \|y - \mathbf{R}x\|^2 \quad (10)$$

$$\lambda^{\overline{ML}} = \min_{x \in x_{j,i}^{(x_{j,i}^{\overline{ML}})}} \|y - \mathbf{R}x\|^2 \quad (11)$$

Where $\mathbf{x} = [x_i, x_{i+1}, \dots, x_{n_T}]$ is a vector of symbol set, n_T is the dimension of the signal transmitting end. When i is one, x_i is the vector of candidate set. d_i is defined as Partial Euclidean

Distance (PED) and $|e_i|^2$ is defined as Distance Increment (DI).

The calculation of λ^{ML} in the formula (10) and (11) is equivalent

to the calculation of Euclidean Distance $d(x) = \|y - \mathbf{R}x\|^2$, and the calculation of $d(x) = \|y - \mathbf{R}x\|^2$ can be expressed as:

$$d_i = d_{i+1} + |e_i|^2, \quad i = n_T, n_T - 1, \dots, 1 \quad (12)$$

Where d_{n_T+1} is initialized and made to equal to zero. Distance increment of the i layer:

$$|e_i|^2 = \left| y - \sum_{j=i}^{n_T} R_{i,j} x_j \right|^2 \quad (13)$$

There is a detailed analysis for the STS algorithm in the reference [1]. The reference [2] has proposed the STS algorithm based on QR decomposition, and use QR decomposition algorithm to obtain the distance between the decoded signal vector and the received signal vector to initialize λ^{ML} . The core idea of the STS algorithm is to start from a node, and do selective searching. If reach a certain

sub-tree, to continue to search down along this subtree, λ^{ML} can be updated or the corresponding $\lambda_{j,i}^{ML}$ in one or several bits may be updated, otherwise abandon the current searching path. The implementation flow chart of the STS algorithm is shown in the figure 2, and the specific implementation steps are as follows:

A unitary matrix Q and an upper triangular matrix R are obtained by performing QR decomposition on H .

Possible symbols of each layer and the Euclidean distance (dist_initial) between the decoding grid point and the receiving symbols can be calculated using the QR decomposition algorithm.

Initialization of parameters: Lambda_ML represents the shortest distance between all grid points that have been searched and the received signal, lambda_ML is initialized to dist_initial (lambda_ML = dist_initial). The lambda_C vector can be used to represent LLR value of each bit, and initialize LLR values of all bits, which means lambda_C is assigned to infinity. Beginning searching starts from the largest layer, namely, set the current layer to the maximum layer.

Solve the distance increment Dist of the current layer, if the current layer is not the first layer and the Dist < lambda_ML, then the first

layer is searched; if Dist >= lambda_ML, then start the search of the maximum distance increment corresponding to each bit, namely lambda_CH_max. Determine whether the Dist is greater than lambda_CH_max, if more than, then this search path is over, go back to the last layer to continue the search, if less than, then continue to search the next layer.

If the current layer is the first layer, when the Dist > lambda_ML, update the maximum value lambda_CH, when lambda_CH > the Dist and the Dist > lambda_ML, search the first layer, otherwise judge the next symbol. Then judge the first sign of the first layer, when the Dist < lambda_ML and the bit corresponding to the current symbol and the bit of the comparison sign are anti bits, update lambda_CH, otherwise update lambda_ML; when the Dist > lambda_ML and the Dist > lambda_ML, if the bit corresponding to the current symbol and the bit of the comparison sign are anti bits, update lambda_CH, otherwise search for the next symbol, when the search of the second layer symbol is over, go back to the second symbol of the last layer to continue to search.

the soft information corresponding to each bit of the ML_symbol_i symbol of each layer is calculated after the search.

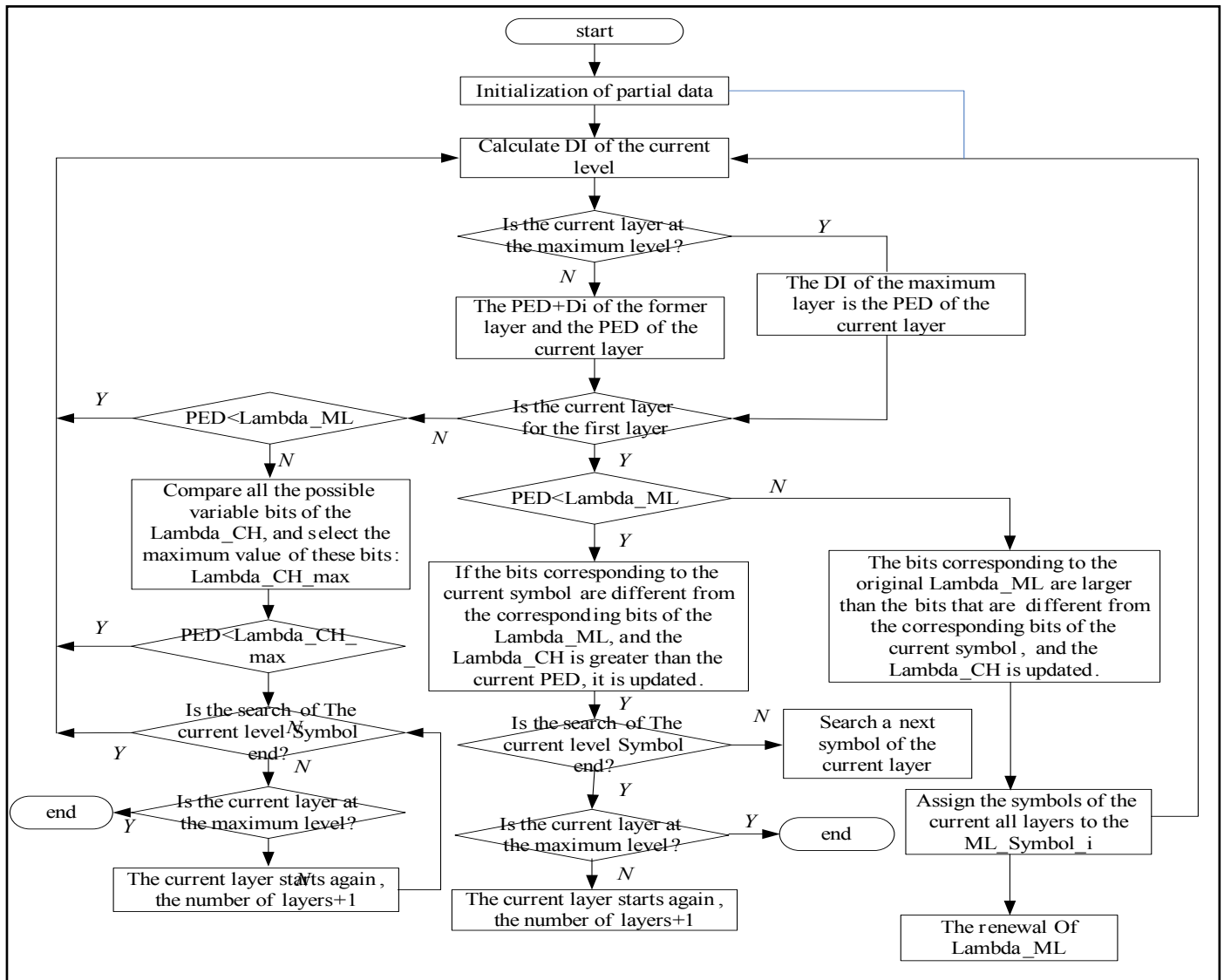


Fig. 2: the implementation flow chart of the STS algorithm

IV. Principle of the preprocessing STS algorithm

From the above analysis, in the STS algorithm the X_M is firstly determined in the process of solving the signal, then X_{M-1}, X_{M-2}, \dots and X_1 are decided in sequence, but the most efficient solving order cannot be guaranteed in accordance with this solving order. In this paper, A preprocessing STS algorithm is proposed, which performs ranking-order rearrangement on columns of H to guarantee the optimization of decoding order. The essence of the algorithm is to rearrange each column of the matrix H before the matrix is decomposed, so that each dimensional signal at the receiving end are searched following a certain strategy instead of being searched blindly from bottom to top or from left to right.

Assuming that each column of the original matrix H is $I = [1, 2, 3, \dots, M]$, where M is the dimension of the matrix, and $J = [i_M, i_{M-1}, i_{M-2}, \dots, i_1]$ is the rearrangement order, there is a correspondence between the I and J :

$$J = IP \quad (14)$$

among them, $P = [e_{i_M}, e_{i_{M-1}}, e_{i_{M-2}}, \dots, e_{i_1}]$, e_{i_i} is a unit column vector of $M \times 1$, the elements of the i_i layer are 1, and the elements of the remaining layers are 0.

The overall flow chart of the STS algorithm is shown in the Figure 3.

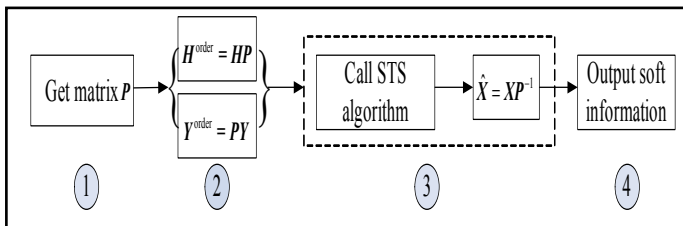


Fig. 3: The overall flow chart of the STS algorithm

The first step in the figure 3 is to get the matrix P . In this section, the ZF algorithm is used to get the initial solution vector, and then the P matrix is obtained based on the principle of maximum probability cutting tree, then its columns are rearranged, the steps are in detail as follows:

The ZF algorithm is used to solve X_{ZF} , where H^+ is the pseudo inverse matrix of the H .

$$X_{ZF} = H^+ Y \quad (15)$$

Calculate the distance between the X vector and the transmission grid points, and sort from the short to the long by the distance:

$$\|X_{ZF}(L) - X_{(1)}\| \leq \|X_{ZF}(L) - X_{(2)}\| \leq \dots \leq \|X_{ZF}(L) - X_{(2^r)}\| \quad (16)$$

$L \in \{0, 1, \dots, M\}$ in the formula (16). As seen from the above formula, $X_{(1)}$ is the most likely solution of $X_{ZF}(L)$, and $X_{(2)}, X_{(3)}, \dots, X_{(2^r)}$ are likely to be cropped. Therefore, the largest dimension of $\|X_{ZF}(L) - X_{(2)}\|$ is firstly decoded, because the Euclidean distance between the signal of this dimension and the transmission signal is relatively large long, which means that it is possible that the search paths of this dimension and below are cut off, so that it can be as soon as possible to eliminate unnecessary search nodes, and to maximize the entire search tree cutting.

Solve $\|X_{ZF}(L) - X_{(2)}\|$ of each dimension:
 for $L = 1: M$

$$d(L) = \|X_{ZF}(L) - X_{(2)}\|$$

end

Sort $d(L)$ from the large to the small:

$$[i_M, i_{M-1}, \dots, i_1] = \text{sort}(d) \quad (17)$$

Thus the P matrix is obtained:

$$P = [e_{i_M}, e_{i_{M-1}}, e_{i_{M-2}}, \dots, e_{i_1}] \quad (18)$$

$Y^{order} = PY$ The H matrix left premultiplies P matrix to obtain. The Y vector postmultiplies P matrix to obtain. The H matrix postmultiplies the P matrix, which means that the H matrix is carried out column vector transformation, and the Y matrix premultiplies the P matrix, which means that the Y matrix is carried out line vector transformation. The STS algorithm can be decoded according to the maximum possible cutting criterion after preprocessing.

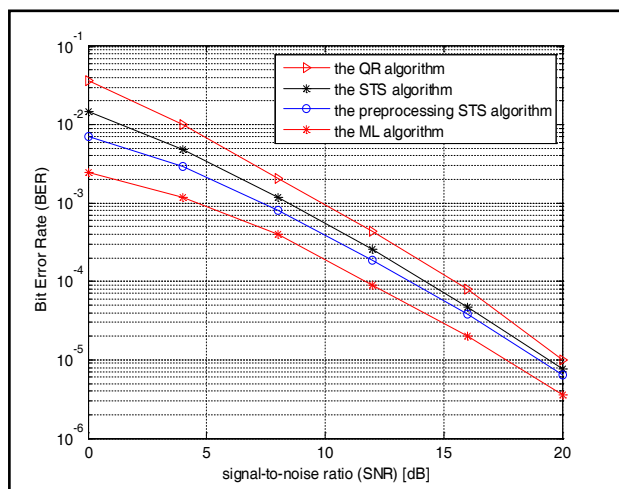
V. Performance analysis of the STS algorithm and the preprocessing STS algorithm

Simulation parameters of the STS algorithm and the preprocessing STS algorithm are shown in the table 1. The code of the STS algorithm and the preprocessing STS algorithm can be written through MATLAB2010a, and then be added to the downlink, because the soft output detection is combined with the Turbo decoding, so the Turbo decoding module needs to be added. The performance and computational complexity between the STS algorithm and the preprocessing STS algorithm are respectively compared in the simulation, and the computational complexity is measured by the number of search points.

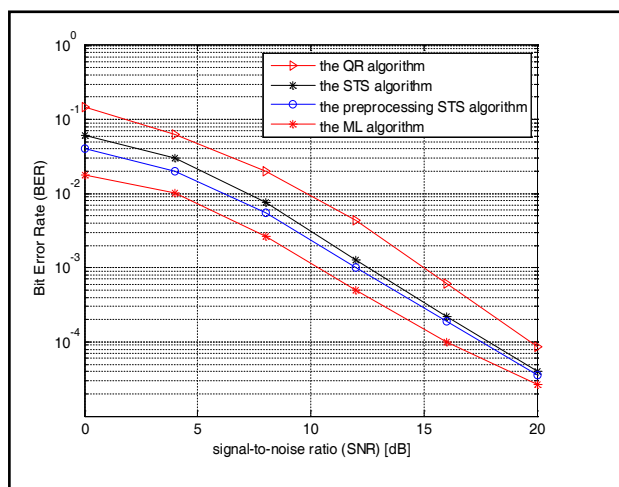
Table 1: the simulation parameters list of two algorithms

Simulation parameters	Value
Antenna configuration	2 x 2 / 4 x 4
Signal detection algorithm	ML/QR/STS/preprocessing STS algorithm
Modulation mode	QPSK
Bandwidth	5M
Channel model	White Gaussian Noise
Wireless channel noise	0dB~20dB

As shown in the figure 4, (a) graph represents the comparison of decoding performance of each algorithm in the 2x2 MIMO system, and (b) graph represents the comparison of decoding performance of each algorithm in the 4x4 MIMO system.



(a) The comparison of decoding performance in 2x2 system



(b) The comparison of decoding performance in 4x4 system

Figure 4: The decoding performance comparison of each algorithm in the QPSK modulation

From the figure (4), it can be figured that the performance of the ML algorithm is the best, and the BER of the STS algorithm and the preprocessing STS algorithm are basically in the same order of magnitude, but the performance of the preprocessing STS algorithm is better than the STS algorithm. In the environment of low SNR, the performance of the preprocessing STS algorithm is improved, but the improvement of the decoding performance is not obvious. For example, compared with the STS algorithm, the performance of the preprocessing STS algorithm is improved 1.5dB at SNR=0dB. The main reason is that the initial solution is obtained by the ZF algorithm, and then the STS algorithm is used to further decode. The decoding performance of the two algorithms are basically the same at SNR=20dB. The maximum of search points in QPSK modulation in the two-antenna system is 16, and the difference between the number of search points in different algorithms is not obvious, so the following is an example based on the four-antenna system. The figure 5 shows the comparison of computational complexity of each algorithm in QPSK modulation, from the figure, it can be obtained that the number of search points of the ML algorithm and the QR decomposition algorithm are fixed, respectively, 256 and 16, which is consistent with the theoretical analysis. Besides, there is a large gap between the STS algorithm and the preprocessing STS algorithm in implementation

complexity, in the case of low SNR, the preprocessing STS algorithm saves more nodes than the STS algorithm. For example, in the case that SNR is zero, the preprocessing STS algorithm saves 50 searching nodes, which saves nearly 1/4 of search points. With the increase of the SNR, the search points of the two algorithms are close to 16 points, which means that the STS algorithm can quickly find the maximum likelihood solution without preprocessing in the high SNR.

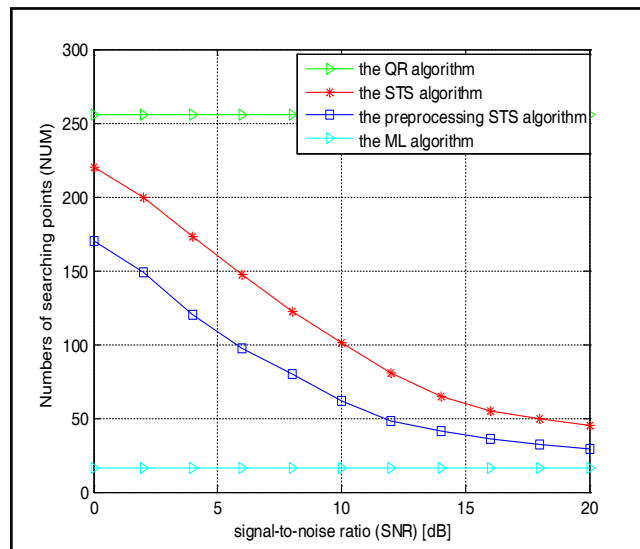


Fig. 5: The comparison of computational complexity in the QPSK modulation of 4 antenna systems

VI. Summary

This paper mainly introduces the soft sphere decoding signal detection algorithm in the space multiplexing mode of MIMO system, first of all, the maximum likelihood detection algorithm, ZF decomposition algorithm, sphere decoding algorithm and soft sphere decoding algorithm are briefly analyzed. Then, according to the shortcomings of the sphere decoding algorithm and the soft sphere decoding algorithm, the preprocessing STS soft sphere decoding algorithm based on the STS soft sphere decoding algorithm is proposed. Finally, Simulation results show that, compared with the STS soft sphere decoding algorithm, the algorithm not only significantly reduces computational complexity, but also improves the decoding performance in the low SNR environment. The algorithm has been applied in the development of TD-LTE-A wireless integrated instruments.

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