

Three-Dimensional Compressed Sensing (3DCS) with Total-Variation for Wide-Band Millimeter Wave Imaging

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Abstract

Modern millimeter wave imaging (MMW) system employs wide-band multifrequency operation to sample a large amount of data to guarantee the image resolution. That means a great burden for data acquisition and processing. Compressed Sensing (CS) provides a theoretical framework to solve the problems like this. Conventional CS used in the image reconstruction always merely considers internal sparse property in a image. While, the MMW system utilize wide-band frequency to scan the target, correspondingly, it may produced a three dimensional (3D) data with multiframe images that contain valuable information, thus we can exactly take the correlations between images into account to acquire more sparse-prior information to contribute for image reconstruction. Based on those conditions, we adopted a effective way of 3DCS with-norm total-variation regularization to recover image from under-sampled 3D data.

Keywords

Holography; microwave; Inverse scattering ; Three-dimensional image processing; Image reconstruction techniques.

I. Introduction

As an effective and harmless imaging technology, millimeter wave image system has been a big hit in the field of concealed contraband detection on the people [1,2]. This system works by transmitting electromagnetic wave and receiving the echoed signal, utilizing the received data and holographic imaging algorithm to inverse the image in the computer, it is a model of near field synthetic aperture radar (SAR) imaging. As we all know, the quality of image depends on resolution, for holographic imaging, the resolution depends on spatial resolution and rang resolution. While, MMW system since the character that the frequency of millimeter waves at the range of 30GHz ~ 300GHz, correspondingly, the wavelengths is about 10mm down to 1mm. Therefore, the short wavelength can capture more spatial information, thus, millimeter wave images can satisfy the demand of high spatial resolution that the detecting image needed [3]. However, a defect of the system is when single-frequency signal wave to scan a target can just focus on a certain distance, that means the objects to be detected will not be focused completely, in the other words, images can't be guaranteed the range resolution. In order to overcome this shortage, the system developed to use wide-band instead of the single-frequency wave to scan the target. Thus, a wide-band wave signal will gather a three-dimensional (3D) data, the data of every dimension can be seen as the corresponding target information in range direction. Therefore, the resolution in range direction rely on the width of frequency [1]. All in all, for the reasons of short wavelength and application of wide-band frequency, MMW system ensure image spatial resolution and rang resolution respectively. However, the high resolution of image obtained on the sacrifice of hardware complexity and data collection. For example, if target with the size 200mm×200mm, bandwidth of scanning is 20GHz, we require to get a 3D date is 200×200×201, that means system need sampling 8.04 million point. Obviously, it is a heavy burden for data collection, storage and transportation. Besides, it is also a great challenge for hardware and scan time. In recent years, Compressed sensing (CS) has been widely applied in the imaging files such as synthetic aperture imaging (SAR), magnetic resonance imaging (MRI), and terahertz imaging to solve the problems of signal

acquisition. According to the CS theory, it can briefly summered :as well as the signal is compressible, by acquiring the prior information in a sparse domain, CS algorithm can recovery the signal as a high accuracy even though the data at a much lower acquisition rate as generally collected by the Nyquist theorem [4]. Therefore, it can definitely improve the accuracy of the recovery signal if we can get more prior information. For a single frequency image recovery, it always considers the sparse characteristic inside the image as the prior information. There have been some applications of CS in the

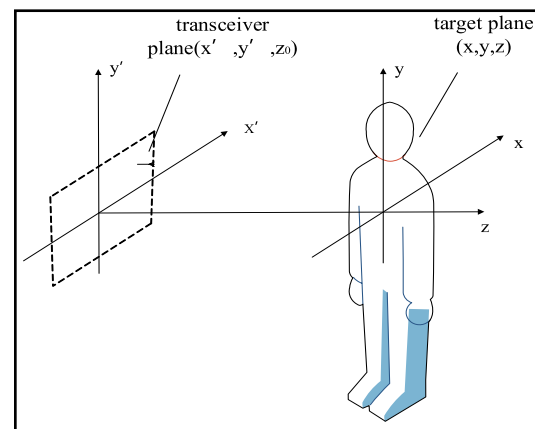


Fig. 1: Imaging system configuration

millimeter field [5], in [5], it is applied in the single-frequency image recovery. Our work are aimed to applied the CS into wide-band frequency operation. Wide-band MMW system received a 3D data, the sparsity also existed between two adjacent dimension data, so we added this part of information to improve the image quality. Under this theory, considered to apply a CS method with 3D total variation (3DTV) into the traditional millimeter wave imaging algorithm, where TV (total variation) [6] is one of the common methods in image processing, in the CS, it is used for signal sparse, we will discuss it in the part 2. Finally, by using nonlinear conjugate gradient [7] for about 200 iterations to solve

a ℓ_1 -norm optimization problem to get high accuracy recovery image and setting the acquisition rate at 50% to 20%.

$$\delta_z = \frac{c}{2B} \quad (3)$$

II. THREE-DIMENSIONAL HOLOGRAM IMAGING ALGORITHM

In paper [1], it has detailedly introduced the theory and algorithm of millimeter wave image in the case of single-frequency and multifrequency. We mainly introduced the wide-band 3D image reconstruction algorithm. The typical measurement configuration is illustrated in fig.1. Transceivers are assumed to be at the planar of (x', y', z_0) , and point on the target can be present as (x, y, z) , if the target can be expressed by the reflected function $f(x, y, z)$, the received echo signal is $s(x, y, w)$. Therefore, the relation between transceivers and target is given by

$$f(x, y, z) = FT_{3D}^{-1}[FT_{2D}[s(x, y, w)]e^{-j\sqrt{4k^2 - k_x^2 - k_y^2}Z_0}] \quad (1)$$

Where $k = \omega/c$ is the wavenumber, ω is temporal angular frequency, and c is the speed of light, k_x and k_y are spatial wavenumber. The $FT_{2D}\{\}$ and $FT_{3D}^{-1}\{\}$ are the 2D and 3D Fourier-transform operators respectively. The exponential term can take the function of focusing, with which the millimeter wave scatter model is suitable for close-field imaging. We can finally get the target image by calculating the amplitude of complex valued reflected function f .

Image resolution is closely related to wavelength and frequency. The planar graph of signal coverage is shown in fig.2, it includes the cross-range resolution and range resolution. For cross-range resolution, we interpret it as the resolution on x - and y - direction or spatial resolution, and it depends on a lot of factors: wavelength λ , aperture D , and the distance between target and antenna. It has two states:

$$\delta_x \approx \frac{\lambda_{mid}}{4\sin(\theta/2)} \Big/ \theta = \min\{\theta_a, \theta_t\} \quad (2)$$

where λ_{mid} is the wavelength of center frequency, θ_a is the full beam width of the antenna, θ_t is the angle subtended by aperture, it is represented as $\theta_t = 2 \arctan(D/2R)$. The cross-range resolution related to bandwidth that is defined as $B = k_{max} - k_{min}$, where k_{max} and k_{min} are the wavenumber at the high and low frequency respectively, so the range resolution influenced by width is expressed as

the 3D data can be seen as the superposition of multiple single-frequency scanning data. By adding the range dimension to ensure the complete spatial details.

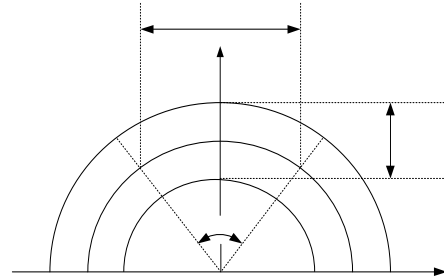


Fig.2: Illustration of signal covered in x -, y - and z - direction

III. 3DCS METHOD FOR THE MMW IMAGING

Compressed sensing became a big hit when it was proposed, for it breaks the traditional Nyquist theorem in signal sampling. With it, we can solve a lot of problems in terms of signal acquisition and cost of hardware. The core idea of compressed sensing is to recover the original signal as high accuracy as possible from much less sampling than that Nyquist needs by exploiting the sparsity prior information [8]. It can be expressed as follows:

$$\arg \min_s \|As - y\|_2^2 + \lambda \Psi^T s \quad (4)$$

where $s \in C^N$ is the original signal, $\Psi^T \in C^{N \times N}$ is the sparsifying transform that the signal in this domain is k -sparsity, such as Fourier and Wavelet transform are usually used in the field of image compression. In CS, the sparsity of signal is a critical factor for the recovery. $A \in C^{M \times N}$ is the undersampled operator, with which we acquire the valuable small amount data y , where $y \in C^M$ ($M \ll N$).

The expression of target and the echo signal has been given in the equation (1), to better apply into CS, it can be briefly written as

$$s = Tf \quad (5)$$

where T is a whole transform operator of equation (1) that can be defined as $T = FT_{3D}^{-1}[FT_{2D}[\bullet]]e^{j\sqrt{4k^2 - k_x^2 - k_y^2}Z_0}$. In the actual

measurement ,it can achieve undersampling by controlling the signal emission.In mathematics,we express the procedure of undersampling pattern though multiplying a mask M ,thus ,the signal we adopted is

$$y = M \cdot s = M \cdot Tf = Af \quad (6)$$

there $A = M \cdot T$ can be seemed as a undersampled operator. M is a same type matrix with S that contains only 1 and 0,then,by controlling the rate of 1 to achieve different sampling rate.

One of the essential parts of CS is the signal sparsity $\Psi^T S$ in equation(4),we should always choose an appropriate sparsifying method as much can ensure the signal sparse as possible,at the same time, keep more valuable information.There are many methods of sparse,such as:Wavelet,total-variation and some joint sparse.In this work,considered utilizing the information between two adjacent images,we adopted total-variation as a sparse domain.TV regularization is usually used in the field of image processing because its better performance in preserving image edges and smoothing noise[9].Two common forms of methods for sparse design are ℓ_1 -norm and ℓ_2 -norm in imaging processing field.In[10], ℓ_1 minimization was proposed as a method in statistics for sparse model section,and those years ℓ_1 -norm has come in emerging field of Compressed Sensing. ℓ_1 -norm and ℓ_2 -norm are defined as:

$$\ell_1: \quad \|x\|_1 = \sum_{n=1}^N |x_n| \quad (7)$$

$$\ell_2: \quad \|x\|_2 = \left(\sum_{n=1}^N |x_n|^2 \right)^{\frac{1}{2}} \quad (8)$$

While ℓ_1 -norm is more suitable for exact sparse signal recovery, whose resolution of optimization problem is sparse.The solution tends to generate more zero and very small residuals,it is less than that ℓ_2 -norm and it more often is the optimal solution.However, ℓ_2 -norm solution tends to have large residuals,more inclined to a local optimal solution.We just can

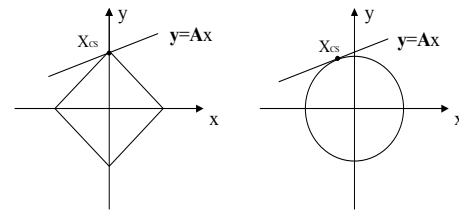


Fig.3: Minimization of ℓ_1 -norm and ℓ_2 -norm

simply analysis the optimal solution from fig.3, in order to facilitate visualization, we consider the case of two dimensions.

the constraint range of ℓ_1 -norm has corners,it is very likely that objective function meet the range at one of the corners where is sparse.In contrast,the ℓ_2 -norm meet-point of would set on anywhere,thus,it has a low probability on the sparse point.So many paper chose TV to deal with the sparsity .

In our work we used ℓ_1 -TV,the model of 3DTV is given by

$$TV_{3D}(f) = \|\nabla_x f\|_1 + \|\nabla_y f\|_1 + \|\nabla_z f\|_1 \quad (9)$$

Where $\nabla_x f$, $\nabla_y f$ and $\nabla_z f$ are the discrete gradient on the x , y, z direction respectively ,they can writtern as:

$$\begin{aligned} \|\nabla_x f\|_1 &= \sum_{i,j,k} |f_{i+1,j,k} - f_{i,j,k}| \\ \|\nabla_y f\|_1 &= \sum_{i,j,k} |f_{i,j+1,k} - f_{i,j,k}| \\ \|\nabla_z f\|_1 &= \sum_{i,j,k} |f_{i,j,k+1} - f_{i,j,k}| \end{aligned} \quad (10)$$

Equation (4) is usually solved by using the ℓ_1 -norm regularization to minimize its Lagrangian form.Thereby the 3DCS model for the millimeter wave image system in our work can be written as

$$\arg \min_f \|Af - y\|_2^2 + \lambda TV_{3D}(f) \quad (11)$$

where λ is a regularization penalty that take the balance between data consistency and image sparsity.More concretely,for the data generated by the wide-band wave system, it can be seen as superposition of each matrix generated by single-frequency wave. So f is formed by $f_1, f_2 \dots f_n$.Therefore, equation (12) can be written as

$$\arg \min_{f_n} \left\| \begin{matrix} Af_1 - y_1 \\ Af_2 - y_2 \\ \dots \\ Af_n - y_n \end{matrix} \right\|_2 + \lambda TV_{3D}(f) \quad (12)$$

Through this CS model we can reconstruct every image that produced by every frequency. This can be better suitable for engineering practice. we adopted the standard nonlinear conjugate gradient method to solve this optimization problem.

IV. EXPERIMENTS AND RESULTS

In the experiment, we scanned a gun to simulate the scenarios of concealed weapon detection. The target plane is parallel to the scanning aperture with the distance at 0.33m, fig. 4 (a) shows the real scene photograph. Data is produced by imaging system operates with the frequency from 130GHz~150GHz scanning an area of 200mm × 200mm, where the center frequency is 140GHz, the length of aperture is 0.1m and scanning interval in plane is 0.001m, that means it scanned 200 × 200 points every frequency, Z-direction wave number is 201. Finally got the discretization data of 200(x) × 200(y) × 201(f). In traditional imaging pattern, there need uniform-spaced full sampling conforms to the Nyquist theorem, and the image recovered by conventional algorithm is shown in fig. 4. When applied compressed sensing, the pattern of sampling is shown in fig. 4(b), they are the show image of mask.

We scanned a real scenario and get a sets of data with a high resolution hologram image to examine the performance of the 3DCS model with 3DTV. The structure is based on the system without high power amplifiers, as the figure showed we scan a real gun to simulate the contraband. And the optical image has shown in fig. 4(a), the recovered holographic image are shown in fig. 4(c). For the sampling pattern, we made the mask to simulated uniform random undersampling, just as (b) shown, where the white represent the point we sampled, by controlling which we can realize different sampling rate, in our work, the sampling rate at 50%, 33%, 25% and 20% for two set of data. Through a series of comparison of recovered image that from undersampled data based on Compressed Sensing, the 3DTV can show different effect for different resolution. Beside we also

made a experiment of CS with the spares prior information only considered the sparsity inside a single image (2DTV) to make a comparison with the method that we proposed.

Equation (14) is calculated by the method of nonlinear CG for about 200 iterations, all the compute were implemented using MATLAB 2015b (x64) on the computer that is windows 7 operation system with the Intel i5-2300 CPU and 8GB memory, the time of 200 iterations calculation need about 5000s.

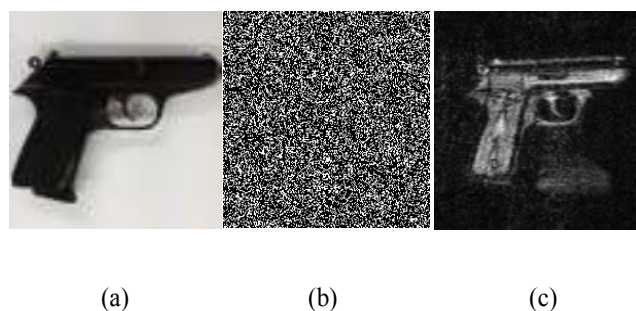


Fig. 4: patterns. (a) realistic setting (optical image). (b) uniform random sampling with the rate at 33%. (d) holographic image

In order to compare the recovery results of different sampling model we choose relative error and peak signal-to-noise ratio as a standard to make a quantitative analysis. Besides, the reconstructed image at every sampling rate can provide a qualitative comparison as the figure 4 showed. Both of the comparison value take the recovered image that full sampled as the reference. Where the the PSNR value of the reconstructed images is computed as

$$PSNR = 20 \lg \frac{I}{\sqrt{MSE}} \quad (13)$$

Where I is the maximum intensity of the the reference image, MSE is the mean square error between the reference and reconstructed output images.

Experimental results analysis

The reconstructed results using two different operation method has shown in and the contrast of 3DCS based on 3DTV between 2DTV has shown in fig. 5 and fig. 6. Where (a) shows the reconstructed images from used 2DTV method. When the sampling rate at 50% even 33% recovery quality is very

nice, and it is difficult to distinguish differences in the visual. Even though the data is just left 25%, it still reserves a main outline of a gun, just a detail information is a little vague, it can completely meet the needs of human recognition. It has

proved that the Compressed Sensing has a great effect on the undersampling reconstruction for hologram imaging. In fig.6, we used the method -3DTV we proposed in this paper. It shows the method that adding the sparse information of the third

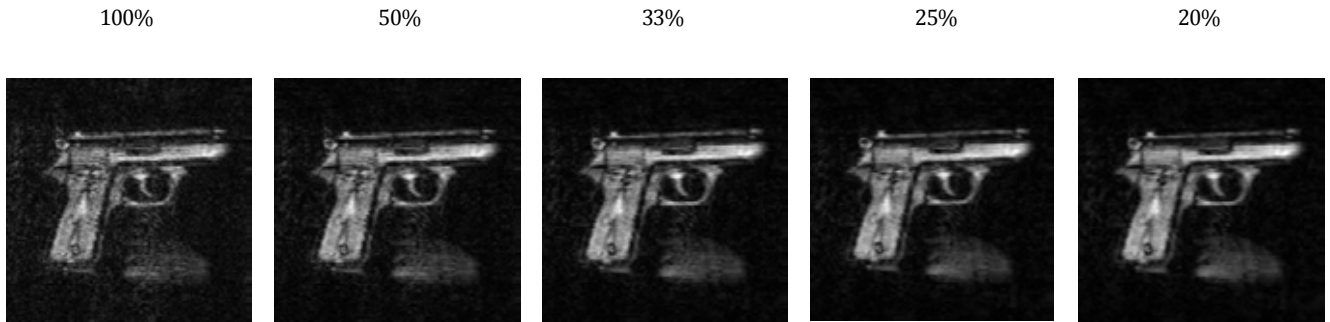


Fig.5: Reconstructed image of 2DCS method on the sample rate from 20% to 50%.

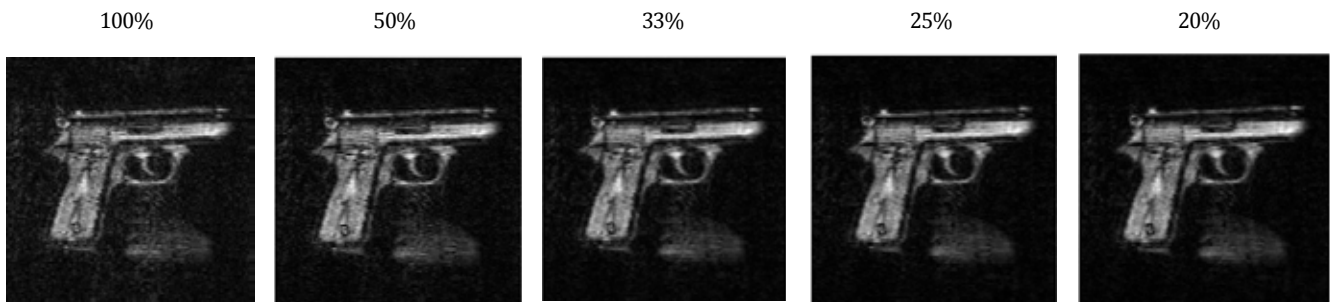


Fig.6: Reconstructed image of 3DCS method on the sample rate from 20% to 50%.

dimension can really improve reconstruction effect. In order to make a more precise comparison, the reconstructed error and PSNR of various undersampling are plotted in fig.7, it can be seen that when the sampling rate at 50% even to 30%, the method of adding sparse information that between images (3DTV) can improve the reconstruction quality compared with 2DTV method, however, the promotion is not so much when sampling rate under 20%.

and 3DTV.

V. CONCLUSION

In this paper, in order to solve the problems of data acquisition in millimeter wave image for security inspection, we proposed the method of 3DCS base on 3D total-variation by adding the prior information of the third dimension to reconstruct image and get a good result. Compared with 2DTV, there is a significant improve of reconstruction effect. Besides, for high resolution original image, the reconstruction can still guarantee great quality at low sampling rate. It was proved that even at 30% sampling rate, the recovered images can meet the requirement of visual recognition no matter the quality of original image. All in all, 3DCS with 3DTV is an efficient method to reduce data acquisition, at the same time, ensure image resolution. It is very suitable for application of MMW imaging system.

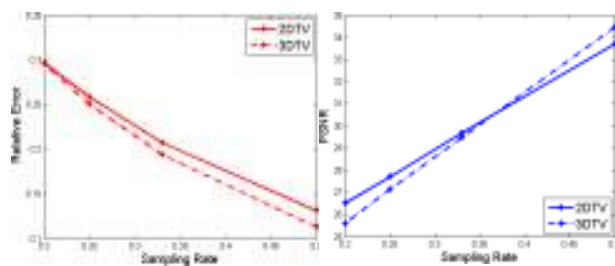


Fig.7: Comparison of error and PSNR with the method of 2DTV

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